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SYMPOSIUM: SUBDWARF STARS

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Trigonometric Parallaxes of Subdwarfs

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TO ascertain the reality of subdwarf character in the H-R diagram, recall that a 10% error in the parallax corresponds to an error of 0.2 magnitude in the absolute magnitude. The added likelihood of systematic errors makes it rather difficult to decide for parallaxes below 0".1 whether the star is a subdwarf or "normal" main-sequence star.

Among stars nearer than five parsecs, there seem to be three clear cases of subdwarfs using absolute magnitude as sole criterion. (Table I). For two classical examples of more distant subdwarfs see Table II. The only parallaxes available for Wolf 134 and Wolf 1037 are based on a very limited number of exposures taken with the 100-inch reflector. Traditionally, reflectors have been considered unsuitable for photographic determinations of trigonometric parallax; there are reasons to believe that the results may be subject to serious errors. On the other hand, there are indications of limitations in the accuracy reached in photographic parallax determinations made with refractors due to systematic errors which may be traced to coma, imperfect focus and other sources. With either instrument, of course, systematic errors may be reduced by using a small configuration of reference stars or by reducing the aperture. A prime requirement is the extension of parallax determinations toward stars of fainter magnitude than the thirteenth, the present practical limit reached with the Allegheny refractor. It is doubtful at this moment

whether longer exposures will appreciably increase the limiting magnitude which we may be expected to reach. We may only surmise what image-tube techniques applied to existing telescopes may eventually bring us. Meanwhile, the most promising solution for obtaining accurate parallaxes of fainter stars lies in the building of a large reflector with a quartz mirror as recommended at the Conference on the Cosmic Distance Scale (*Astron. J.* 63, 150, 1958). A 60-inch reflector would permit the photographing of stars down to the eighteenth magnitude. The building of such a large reflector specifically designed for astrometric work is certainly one of the most important desiderata toward obtaining parallaxes of faint stars, and will also benefit the study of subdwarf stars.

DISCUSSION

SCHWARZSCHILD: I wonder whether we might ask Dr. Strand to tell us what the practical magnitude limit

TABLE I. Subdwarfs using absolute magnitude as sole criterion.

	Vis. mag.	Sp.	Proper motion	Parallax	Deviation from main sequence
τ Ceti	3.6	G4	1".92	$0".275 \pm 0.005$	1.0 mag.
Gr 1618	6.8	K5	1.45	$0".222$	5 1.5 mag.
Kapteyn's star	9.2	M0	8.79	$0".251$	7 2.5 mag.

TABLE II. Two classical examples of more distant subdwarfs.

	Pg. mag.	Sp.	Proper motion	Relative parallax	No. of exposures
Wolf 134	15.2	K3	1".74	$+0''.052 \pm 0.007$	14
Wolf 1037	14.5	K4	1.57	$+0.054 \pm 0.004$	16

will be for the 60-inch astrometric reflector he is contemplating at present, assuming that it would be set up in a particularly favorable location.

STRAND: May I first say a word about using conventional reflectors for astrometric purposes. I do not look too hopefully on using, for instance, the 100-inch reflector for determining trigonometric parallaxes because when you want to get ultimate precision, the conventional reflector has certain disadvantages. For instance, there is the temperature effect. In order to get good parallaxes you have to observe the stars at large parallax factors. However, the early evening is always bad with reflectors of the type of the 100-inch, because it takes some time before the mirror settles down. A telescope used for parallax determinations should have optics which are rather insensitive to temperature effects and, therefore, in the telescope that we have proposed for astrometric work we are hoping to use quartz optics which are now feasible.

With a 60-inch aperture we should get down to about the eighteenth magnitude in about seven minutes.

I would like to mention something else about the

subdwarfs. It is extremely difficult to get their parallaxes as Dr. Van de Kamp has explained. In 1955 at a Dublin meeting I was authorized by Commission 19 to make up a list of subdwarfs and recommend them to various observatories with parallax programs. Most of these stars had parallaxes in the Yale catalogue, but all these parallaxes were of the old-fashioned kind where there were perhaps 16 plates, or in some cases more, and the parallaxes therefore had very little weight. I made up a list with about 20 stars and I requested that at least three observatories pick the same stars out of this list. Now, with 20 stars and 50 plates each as I requested, that meant a thousand plates completing the whole program. This is very much, especially when you have stars which are quite faint on conventional reflectors. Now I have recently sent inquiries to find out how this program has progressed and I am sorry to say that the progress has been fairly limited with the exception of one observatory, our institution here at Pittsburgh. So we have been aware of this problem of subdwarfs for many years and in spite of much effort in this difficult field we have as yet not turned out good material to establish their absolute magnitudes.

SCHWARZSCHILD: I for one am enormously happy that a major technical effort is going into driving accurate parallax measurements to fainter magnitudes because I think that the subdwarfs, just as the very faint normal dwarfs, are so important astrophysical items that this alone will make this effort very much worthwhile.

Space Motions of the Subdwarfs

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It could easily be true that each participant in this symposium—and most of the audience—has a different concept of our subject. What is a subdwarf? By definition, or rather by common usage, it is a star that is below the main sequence far enough to be outside the inevitable observational scatter but not far enough to be called a white dwarf. What, then, is the main sequence? Again by common usage it is the sequence in the H-R diagram that is populated by most of the nearby stars for which good trigonometric parallaxes have been obtained. However, astronomical literature contains many references to subdwarfs for which no trigonometric parallax is available. Some of these are only known to have high velocity, while others are only known to show weak metallic lines in their spectra. On the other hand, when we examine the luminosities of some 200 stars for which fairly reliable trigonometric parallaxes are available, there are less than two dozen which fall farther below the main sequence, defined by the remainder, than can be accounted for by uncertainties in the parallax. And of these “trigonometric” subdwarfs, only two or three show weak metallic lines in their spectra and have space velocities in excess of any that defines the main sequence.

Before talking of subdwarfs it is therefore necessary to strictly define a dwarf, or a main sequence. From accurate photometry of bright galactic clusters such as the Hyades and Praesepe it has been known for many years (Eggen 1950) that the main sequences for cluster stars later than about F0 represent almost a one-to-one relation between luminosity and color—the few stars in each cluster that violate this relation, usually in the sense that they fall above the sequence defined by the remaining stars, are explainable as unresolved double or multiple systems. However, only for one cluster—the Hyades—do we have an accurate means of determining the distance.

Let us then define the “main sequence” as the dwarf sequence of the Hyades stars in the $(M_V, B-V)$ plane. As mentioned above, there are only a few nearby stars with trigonometric parallaxes sufficiently accurate to definitely place them below this main sequence. Furthermore, photometry shows that most of these stars have an ultraviolet excess when compared with the main-sequence stars in the $(U-B, B-V)$ plane. This excess is empirically related to the displacement below the main sequence. The excess and the displacement are also related theoretically through the line-blanketing theory (Sandage and Eggen 1959). The blanketing theory is based on considerations of the effect of metal content on the observed colors—the lower the metal content the weaker the lines and, therefore, the bluer the star for a given temperature. If we accept this view,

then we give status to the “subdwarfs” mentioned above for which we know only that the metallic lines in the observed spectra are weak. In this connection it is fortunate that the stars in the cluster which defines our main sequence—the Hyades—show as strong a metallic spectrum as any star observed near the sun. There remains, however, the question of the few parallax stars that show both a definite displacement below the main sequence and an ultraviolet excess without having been noticed as showing a weak metallic spectrum. It is assumed—and this assumption may represent one of the weakest steps in the argument—that for stars very little affected by line-blanketing and therefore very little displaced from the main sequence, the ultraviolet excess is a more sensitive criterion of “subdwarfiness” than the appearance of the spectra. The converse is probably true for stars having extremely weak metallic lines, such as Groombridge 1830 (near the sun) and the members of halo globular clusters which all show about the same amount of ultraviolet excess, $\approx 0^m.25$, but with obvious differences in the strength of the metallic lines in the spectra. For example, NGC 6397 (Eggen 1960), ω Centauri (Eggen 1960) and M13 (Baum, Hiltner, Johnson, and Sandage 1960) all show the same ultraviolet excess, but the first falls in Deutsch’s (See Kinman 1959) category C (very weak metallic lines) the second in category B (weak metallic lines), while the third is in category A (nearly normal metallic lines).

Therefore, for the present purposes, in addition to those few stars whose trigonometric parallax places them below the main sequence, we will accept as subdwarfs those stars showing either weak metallic spectra or an ultraviolet excess. Because of the observational difficulties involved in obtaining accurate ultraviolet magnitudes, we will also require that the observed excess be larger than $0^m.05$.

There are approximately 100 stars in the literature that have been classified as subdwarfs on the basis of their weak metallic lines (Eggen and Sandage 1960). From published three-color observations of about a third of these, plus unpublished observations with the 20-inch reflector at Mt. Palomar (Sandage 1960) or the 24-inch Cape refractor and the 74-inch Pretoria reflector (Eggen 1960) of most of the remaining stars, it is known that with only one or two exceptions they all show an ultraviolet excess with respect to the Hyades main-sequence stars. This is also true of an additional two dozen new southern subdwarfs that have been found by T. J. Deeming and the Cape observers with the 74-inch Radcliffe reflector.

To compute accurate space motions of these stars we of course need accurate proper motions, radial velocities

and parallaxes. It is of interest to examine the situation with respect to each of these requirements.

1. Radial velocities are available for most of the subdwarfs now in the literature. The majority were determined by Joy, with the Mt. Wilson reflectors, or by Popper and Münch, with the 82-inch McDonald reflector. In most cases where an overlap is available, the results from the two observatories are in good agreement and an independent determination by Greenstein (1960) from 200-inch coude plates has strengthened the previous results for several stars. For the southern objects, many new determinations have been made with the 74-inch reflector by Deeming.

2. Although a "crash" program can always be used to obtain radial velocities when the demand becomes strong enough, the same is obviously not true for proper motions. It is therefore unfortunate that the greatest uncertainties in the observed motions of the subdwarfs lie in the presently available proper motions. About a dozen subdwarfs have a relatively long meridian history from which very accurate motions are obtainable and a few, brighter than 9th visual magnitude, fall in those Cape or Yale zones that have a well-documented astrometric history, but the majority of the stars have published proper motions that are virtually useless for determining space motions of the required accuracy. Most of these stars appear on first epoch astrographic plates taken at various observatories around 1900; relatively accurate proper motions can now be obtained by taking repeat plates. This has been done at the Royal Greenwich and Cape Observatories for the Cape, Oxford, Greenwich, and one or two other zones and proper motions are being determined. Excellent sources of good proper motions are the relative values obtained from parallax series, especially when extra, later epoch plates are also taken to increase the time-base. Such proper motions, in both coordinates, have been determined for most of the stars on the parallax programs of the Royal Greenwich and Cape Observatories and for most of the faint Ross stars observed for parallax at the Yerkes Observatory, but this source of proper motions for a large number of faint stars observed on other parallax programs, such as that of the Allegheny Observatory, remains to be tapped. A great amount of work remains to be done in searching out, evaluating and utilizing the scattered astrometric material that now exists for subdwarfs. One heartening point—the proper motions of most subdwarfs are so large that differences between absolute and relative values, or questions of which "fundamental" system to use, are immaterial.

3. First class trigonometric parallaxes, greater than $0''.1$, are available for only four stars that have been classified as subdwarfs on the basis of their spectra—Groombridge 1830 (an extreme subdwarf), μ Cassiopeiae (a mild subdwarf), Barnard's star and, perhaps, τ Ceti (a very mild subdwarf). However, if we accept the blanketing theory and equate the observed ultraviolet excess to

displacement below the main sequence, photometric parallaxes can be obtained for any subdwarf for which accurate three-color photometry is available. An important point should be noted in this connection, i.e. since ultraviolet excesses have not been found among the "trigonometric" subdwarfs of later type than about K0, it may be that the blanketing line and the intrinsic ($U-B, B-V$) line coincide for later types and therefore a sample of stars chosen on the basis of ultraviolet excess will contain no K- or M-type subdwarfs. Other selection effects will also discriminate against such late type stars since the ratio of subdwarfs to main-sequence stars of apparent magnitude, say, ten, among objects with proper motions exceeding $0''.2$ /year is much larger for F- and G- than for K- and M-type stars and so discovery programs have naturally concentrated on the earlier types. The only certain, late-type subdwarfs are Kapteyn's star (HD 33793), which the large trigonometric parallax places well below the main sequence and a small group of sdK and sdM stars, including Barnard's star, discussed by Joy (1947). From a theoretical standpoint there is a question as to whether or not the late-type subdwarfs should show weakened metal lines (cf. Unsöld 1958), but Joy's observational evidence seems to be definitive.

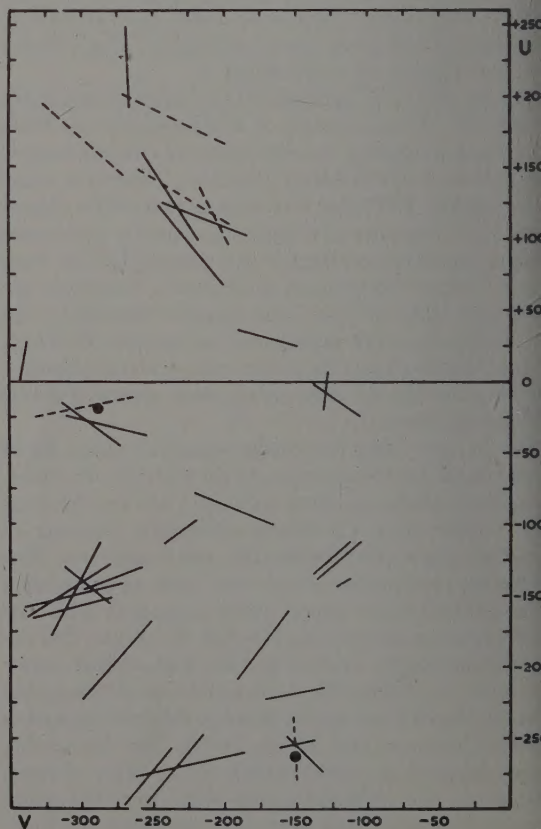


FIG. 1. (U, V)-loci for extreme subdwarfs.

Photometric, astrometric, and radial velocity programs are continuing in an attempt to improve the available data for the known subdwarfs and to discover new members of the class. At present, complete information of good quality is available for only about 50 stars. Of these, 28 have observed ultraviolet excesses greater than $0^m.2$ and should therefore lie near the subdwarf sequence defined by the members of the Groombridge 1830 group and M3 (Eggen and Sandage 1959). In this assumption, photometric parallaxes were obtained which were then used to compute the space motion from the observed radial velocities and proper motions. The space motion vectors are designated U , V , and W and are directed, respectively, away from the galactic center, in the direction of galactic rotation and toward the north galactic pole. The (U, V) -loci for these stars, resulting from the assumption that the luminosity lies within $\pm 0^m.25$ of the subdwarf sequence of M3, as calibrated by the Groombridge 1830 group, are plotted in Fig. 1. The two filled circles represent Kapteyn's star (near the zero of the U -axis) and Groombridge 1830, for both of which well determined trigonometric parallaxes are available. The (U, V) -loci for a few RR Lyrae variables with well-determined proper motion and radial velocity are shown as broken lines; the luminosities were assumed to be $+0^m.5 \pm 0^m.25$. The two stars that form the wide binary HD 134439/40 fall off the diagram at $(U, V) = (-280, -420)$. There are two interesting features of Fig. 1.

(1) If we draw a line through the diagram at $V = -210$, which represents the currently accepted value for the galactic circular velocity at the sun's distance from the galactic center, it will very nearly lie at the center of the V -distribution of the plotted stars. If this is the correct value for the circular velocity, the sample of stars considered here is equally divided between those with direct and those with retrograde galactic orbits. In other words, the system of extreme subdwarfs does not take part in galactic rotation. However, if we assume an erstwhile value near 300 km/sec for the circular velocity, HD 134439/40 is the only system moving in a retrograde orbit.

(2) In spite of the small sample, there are two or three remarkable clumpings in the distribution of (U, V) -loci. The two loci that intersect near Groombridge 1830 are both members of the group previously discussed (Eggen and Sandage 1959) and the variable indicated by the broken line is RR Lyrae. [Photometric data for two additional members of the group has been obtained with the 74-inch reflector since the figure was drawn.] The largest clumping involves six stars that intersect near $(U, V) = (-150, -300)$. It seems reasonable to suppose that insofar as their motions are controlled by the central force of the galaxy, the six stars are moving in isoperiodic orbits.

The remaining 22 stars have ultraviolet excesses between $0^m.15$ and $0^m.2$ and the photometric parallaxes

were computed from the assumption that their displacement below the Hyades main sequence is that predicted by the blanketing theory. The (U, V) -loci, determined from an estimated uncertainty of $0^m.25$ in the predicted luminosity, were plotted (not shown here) and again some clumping of the loci was noticed. The most conspicuous grouping contains four stars that appear to share the space motion of the "trigonometric" subdwarf, μ Cassiopeiae. The center of the V -distribution of these milder subdwarfs is $V \approx -175$ km/sec.

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DISCUSSION

SPITZER: I was very much interested in the physical groups among these high-velocity stars indicated on your slide. What is the physical separation between members of one group?

EGGEN: Most of this material is limited to three or four hundred parsecs just because the stars that we have accurate information for cannot be much further away. The stars in the four or five groups that are indicated on the slide are actually spread all over the sky. Most of them will be at a distance of about 100 parsecs.

SPITZER: It seems hard to believe that you should have a physical grouping of stars with that big distance between them.

EGGEN: This depends entirely on what mechanism you assume for their motions. The whole problem of isoperiodic motions comes into this. The motions are perhaps not as precisely parallel as once thought necessary, but the stars might be in isoperiodic orbits.

KING: The thing that surprises me the most about the existence of groups like this one, if they do turn out to be real, is that encounters with local irregularities in mass, like the large interstellar complexes that Mr. Spitzer has discussed, should not have changed the velocities in a random fashion enough to wash out completely groups of this kind.

SCHWARZSCHILD: I think it is just plain true that theoretically-minded astronomers like myself are surprised and a little worried about this new phenomenon. So the strengthening of the observational material towards proving or disproving this enormously important item would appear much worthwhile.

Interior Models for Subdwarf Stars

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INTRODUCTION

OBSERVATIONAL evidence points to one major physical difference between subdwarfs and ordinary dwarfs, a difference in their chemical compositions. Subdwarfs are metal-deficient in comparison with solar-type stars. Subdwarfs, being essentially main-sequence objects, can be considered to be homogeneous in composition. Recently, Hazelgrove and Hoyle (1959) have also studied the structure of metal-poor stars in a comparison of Population I and Population II main-sequences.

In our investigation, models for stars of masses 0.6, 0.8, and 1.0 solar mass were constructed for several compositions: X , the hydrogen content by weight, varying from 0.999 to 0.75 and Z , the metal content by weight, varying from 0.01 to 0.001. At the beginning of this work, it was not realized that some stars had still considerably lower metal contents. We shall discuss in a following section the effect of metal abundances lower than $Z=0.001$.

THE DEEP INTERIOR

The general structure consists of a core in radiative equilibrium surrounded by a convective envelope, a structure similar to that found by Osterbrock (1953) and Limber (1957) for red dwarf stars and Schwarzschild, Howard, and Härm (1957) and Sears (1958) for the sun.

In the core, the opacity is obtained from the tables of Keller and Meyerott (1955). The source of energy generation is due to the proton-proton chain, both because of the relatively moderate temperatures and the heavy-element deficiency. A family of cores was integrated from the center in the way given by Schwarzschild (1958) with the help of an IBM 650 computer. They were fitted to the adiabatic envelopes of Osterbrock (1953). In this way a family of interior models was obtained, each model characterized by the depth of the convective envelope.

The luminosity of the star depends essentially on the conditions in the deep interior, but the radius depends strongly on the conditions on the outside. Therefore, in order to determine the radius of a star and thus its effective temperature and position in the H-R diagram, the outer boundary conditions must be studied in detail.

THE ATMOSPHERE

The very outer part of the atmosphere is in radiative equilibrium. Grey atmospheres were constructed until

the region of convective instability of the hydrogen ionization zone was reached. The opacity is due to the negative hydrogen ion at lower temperatures and to photoionization of hydrogen at higher temperatures.

It is often assumed that the temperature gradient is practically equal to the adiabatic gradient as soon as convection sets in. Vitense (1953) has shown that in the case of the sun one must take into account the combined effect of radiation and convection transports in the hydrogen convection zone. The temperature gradient is then intermediate between the radiative and convective gradients. This approach, first suggested by Biermann (1945), recently used by Kippenhahn, Temesvary and Biermann (1958), requires the knowledge of the mixing length, or the distance travelled by a convective element before losing its identity into the surroundings. This mixing length ℓ , is of the order of magnitude of the pressure scale height H in the atmosphere.

Under these conditions, the energy flux in the atmosphere can be written

$$F_{\text{total}} = F_{\text{rad}} + F_{\text{conv}}, \quad (1)$$

where F_{total} , F_{rad} and F_{conv} denote the total, radiative and convective flux respectively.

Equation (1) can be written

$$\sigma T_e^4 = \frac{16}{3} \frac{g}{\mathcal{K}} \sigma T^3 \frac{dT}{dP} + c_p \rho v \ell \left[\left(\frac{dT}{dr} \right)_{\text{ad}} - \left(\frac{dT}{dr} \right) \right], \quad (2)$$

where T , P , ρ and $\mathcal{K}$ represent the temperature, pressure, density, and opacity, respectively, at a distance r from the center of the star. T_e is the effective temperature, g the surface gravity, σ Stefan's constant and c_p the specific heat at constant pressure of the material. The average velocity of a convective element is denoted by v and the subscript "ad" stands for "adiabatic."

A comparison between Vitense's solar atmosphere model (1953) and the interior model for the sun by Schwarzschild, Howard and Härm (1957) points to a ratio of ℓ/H equal to 1.5.

The first series of calculations for subdwarfs was made with $\ell/H=1.0$ (Demarque 1960). Recently a new series of models with $\ell/H=2.0$ has been computed for comparison purposes. The results are illustrated in Fig. 1. The calculations were automatic for any given effective temperature, surface gravity and chemical composition.

DESCRIPTION OF RESULTS

The principal results are found in Fig. 2, which is a theoretical H-R diagram (M_{bol} vs $\log T_e$ diagram). The

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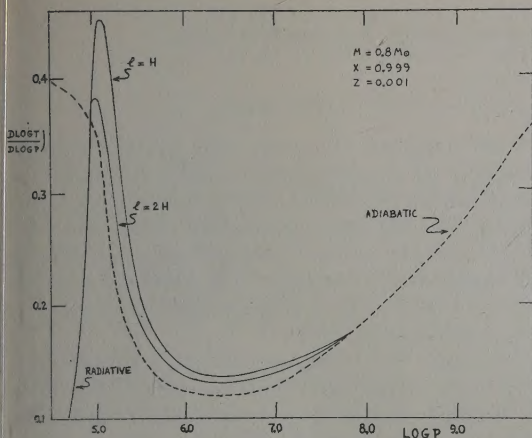


FIG. 1. The effect of the choice of the mixing length ℓ on the structure of the hydrogen convection zone is illustrated. In this sample, for $5.0 < \log P < 8.0$, the value of $(d \log T / d \log P)$ is superadiabatic.

our main-sequences indicated correspond to four different chemical compositions. They are metal-rich ($Z=0.01$) and metal-poor ($Z=0.001$), helium-rich ($X=0.75$) and helium-poor ($X=0.999$ and $X=0.99$). A decrease in metal content, keeping X fixed, from $Z=0.01$ to $Z=0.001$ corresponds to a shift of about 0.3 magnitude in the position of the main-sequence. Similarly, a change in the value of X from 0.999 to 0.75, keeping Z fixed, corresponds to a shift in the position of the main-sequence of about 0.7 magnitude. On each sequence, the circles represent stars of masses 1.0, 0.8, and $0.6 M_{\odot}$, respectively, from high to low effective temperature. It is interesting to note the remarkable spread in luminosity of stars of the same mass, but with different chemical contents, a fact of much interest from the point of view of evolutionary theory.

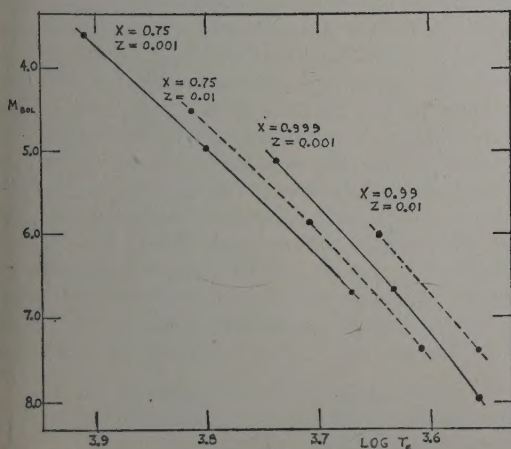


FIG. 2. Relative positions of main-sequences for different compositions.

It also appears that the respective effects of metal deficiency and helium deficiency counteract each other, thus tending to keep metal-poor helium-poor main-sequences close to metal-rich helium-rich main-sequences. One can determine the relationship between the effect of a small variation δZ in the metal content and the equivalent δX in the hydrogen content keeping the position of the main-sequence unchanged. From homology considerations, one finds

$$\delta Z = 0.069 \delta X. \quad (3)$$

Similarly, an homologous transformation enables us to extrapolate to Z values lower than 0.001. One finds

$$(Z+0.01)R^{-5.5} = \text{constant}, \quad (4)$$

which indicates a small sensitivity of the stellar radius R to a very small change in the metal content. Physically, this can be explained by the fact that the

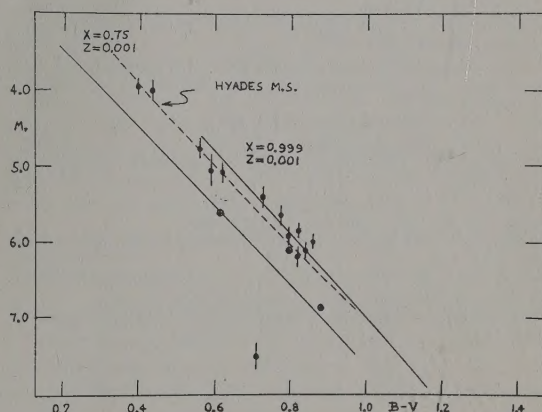


FIG. 3. Comparison with the data of Sandage and Eggen (1959). The symbol $\oplus$ stands for members of the Groombridge 1830 group.

opacity is mostly due to the free-free transitions of hydrogen and helium and not to the bound-free transitions of the metals. It appears, therefore, that a main-sequence of metal-free stars ($Z=0$) would lie very close to a main-sequence of stars with $Z=0.001$.

COMPARISON WITH OBSERVATIONAL DATA

The lack of an accurately determined mass for a subdwarf star is at the moment a most serious handicap. Wyller (1956) has obtained the mass of the components of 85 Pegasi, which unfortunately do not exhibit strong subdwarf characteristics. The situation is still worse in two cases quoted by Eggen and Sandage (1959), ADS 10938, with only three observations since 1900 and ADS 1644, which has an unfavorable inclination near 90° .

Positions in the H-R diagram can nevertheless be compared. Figure 3 shows the results plotted in the

color-magnitude diagram (M_v vs $B-V$ diagram). The points represent observed subdwarfs which have been corrected for their ultraviolet excess, due to weak line blanketing, by Sandage and Eggen (1959). The bolometric corrections of Kuiper (1938) and the colors from Johnson and Morgan (1953) have been used. The dashed line represents the position of the Hyades main-sequence from Sandage (1957). It is interesting to see that subdwarfs then lie on or above the Hyades main-sequence and fall satisfactorily in the area of our models. This result is in good agreement with photometric scanning by Melbourne (1959). The three members of the Groombridge 1830 group seem to lie systematically below the others, suggesting a higher helium abundance.

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DISCUSSION

SCHWARZSCHILD: I cannot help but feel extremely pleased to see an investigation of this nature presenting us with such a substantial body of carefully constructed models. This is in happy contrast with some other recent occasions where we were presented with flow diagrams and similar details of modern computational technique and then had to wait a long time before we got to see actual astronomical results.

One point Dr. Demarque discussed appears to me terribly important, namely his new ideas of how to use his detailed models to possibly estimate the helium content of the subdwarfs. The subdwarfs presumably represent the composition of the galaxy in its early stages. Obviously it would be of very great importance to know whether the galaxy started with a composition of pure hydrogen. If we can trust Dr. Aller, I take it that this problem has not been answered by spectroscopic observations.

ALLER: You can trust what I say in this connection because there is nothing known.

EGGEN: In connection with the models of Dr. Demarque I would like to point out that we do have observations bearing on the masses of subdwarfs. There exists no one good case of a subdwarf in a binary system which you can say with assurance what its mass is. But when you use the inadequate information as we have as possible we consistently come out with smaller masses than we would expect.

Compositions and Mean Atmospheric Parameters of Subdwarfs

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THE most striking characteristic of the spectrum of a G or F subdwarf is that the metallic lines are weak compared with those of a normal star of the same surface temperature. Earlier investigations of HD 19445 and HD 140283, based on plates secured by F. Sanford suggested that these features required a hydrogen/metal ratio of the order of ten times that of the sun (Chamberlain and Aller 1951). Stars which had been classified as A were relegated to late F spectral classes. Subsequent work has shown that these metal deficiencies were actually underestimated and the hydrogen/metal ratio is more nearly of the order of a hundred times greater than the solar value. The Burdges (1956) investigated several stars, finding a range of objects with varying amounts of metal depletion, though no stars appeared to be more extreme than HD 140283. Recently, Baschek (1959) has published an investigation based on plates secured by Unsöld at Mt. Wilson. Miss Roman (1955) has measured subdwarf colors while spectral energy distributions have been measured by Melbourne (1959). The blanketing effect has been studied by Schwarzschild, Howard, and Searle (1955), and most recently by the Burbidges, Sandage, and Wildey (1959) who have shown that when appropriate corrections are made for the blanketing effect the *UBV* colors for HD 19445 are the same as for the sun.

The extreme faintness of these stars renders composition studies difficult; the most interesting objects are of the seventh magnitude or fainter. Furthermore, since the lines (particularly those of ionized metals) are extremely weak, it is necessary to use high dispersions and therefore long exposures with large telescopes. The most extensive high-dispersion studies have been those carried out on coude plates secured at Mt. Wilson and Palomar (Aller and Greenstein 1960). This study embraced the three G-type subdwarfs, HD 19445, HD 140283, HD 219617 and the A-type subdwarf HD 161817. The first two stars were studied in great detail on Palomar coude plates that had a dispersion of 4.5 Å/mm. We used a curve of growth procedure in which the stars were compared differentially with the sun. No attempt was made to obtain the temperature from the line spectrum. Rather, we used Melbourne's (1959) spectral energy scans corrected for line blanketing to obtain the temperatures of HD 19445 and HD 140283. For the other stars it was necessary to use

the *UBV* photoelectric colors. A comparison of the enhanced and neutral lines of calcium, iron, chromium, and titanium then gave the electron pressures. For the sun we assumed a temperature of 5700°K, $\log P_e = 1.30$. The results for the three cool subdwarfs were:

	HD 19445	HD 140283	HD 219617
Temperature	5800°K	5450°K	5900°K
$\log P_e$	0.90	0.34	0.85

Notice that these so-called F subdwarfs are really G subdwarfs, and extreme weakness of the lines produces the appearance of the spectra.

Subdwarfs show a range in composition. As compared with the sun, the metal/hydrogen ratio shows a deficiency of 20 for HD 219617, 40 for HD 19445, and 100 for HD 140283. Baschek finds an even greater metal deficiency than this for HD 140283. He used a model atmosphere and Unsöld's curve of growth.

The deficiency is not the same for all elements. For example, the carbon abundance as derived from the *CH* lines indicates a depletion considerably greater than that of the metals. The logarithms of the depletion factors for carbon are

	HD 19445	HD 219617	HD 140283
(stratification)	+2.55	+2.25	+3.40
(uniform distribution)	+2.25	+1.95	+2.90

The figures in the first row are deduced from a model in which the molecular *CH* is assumed concentrated in the superficial layers, whereas in the last row it is assumed that both the *CH* and metallic lines are formed in the same layers. On the other hand, nickel shows a smaller depletion than does iron. Nucleogenesis arguments have been proposed to account for these differences.

THE INFLUENCE OF DIFFUSION

We must now consider whether the atmospheric abundances in subdwarf stars are affected by diffusion. Thermal and pressure gradient diffusion would both act to cause the heavier elements to sink downwards more rapidly than the lighter elements. Over periods of time comparable with (or greater than) the age of the solar system, thermal diffusion could produce noticeable alterations in abundance ratios for shallow convection zones. The importance of this effect may be guessed from the following calculations carried out for the sun (Chapman and Aller 1960). The entries in Table I give the factors by which the indicated abundance ratios (as obtained for the present solar atmosphere) have to be multiplied in order to get the abundances at the time of the formation of the solar system. It is assumed that the depth, *h*, of the convection zone (expressed in units of the solar radius *R*) has remained the same for

TABLE I. Correction factor to solar abundances.

<i>h</i>	0.30	0.20	0.14
Fe/Si	0.107	1.05	1.19
Pb/Si	1.19	1.27	1.54

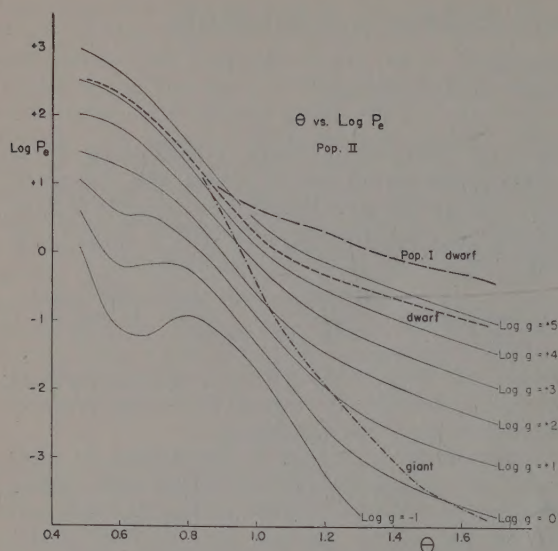


FIG. 1.

the last 4.5×10^9 years. Hence the diffusive depletion will depend critically upon the depth of the convection zone, being very large for shallow layers. One might suppose that if the subdwarf convection zone is very shallow and if the star is very old, diffusion will have played an important role.

On the other hand, if diffusion were to account for all heavy element depletion in subdwarfs, we would ex-

pect a smooth dependence of depletion factor on atom weight. The observational data, although affected by considerable uncertainties do not substantiate this conclusion. The abundance abnormalities are probably produced essentially by nucleogenesis effects. Although diffusion could play an important role in regulating the absolute number of atoms in the atmosphere it cannot affect such observables as the Ni/Fe ratio which appears to be abnormal.

Accurate data on additional stars would be very much worthwhile. In the meantime, the existing material should be analyzed by model atmospheres such as those described by Swihart. What is needed is a series of calculations for a network of model atmospheres to predict profiles and equivalent widths as a function of effective temperature, surface gravity, and metal/hydrogen ratio. For any given star the temperature can probably be deduced from the measured energy distribution. The surface gravity and metal/hydrogen ratio could then be found from a comparison of observed and predicted data. Such a program is well within the range of modern computational resources. It is particularly important to calculate the profiles of the stronger lines by model atmosphere methods. For example, the H_γ profile calculated with Swihart's earlier (1956) model atmosphere and with the Kollbath-Griem theory (1959) gives a fairly good agreement with observations for HD 140283, but there are indications of extremely broad wings. From $\lambda 4000$ to the Balmer limit, overlapping wings of the hydrogen line

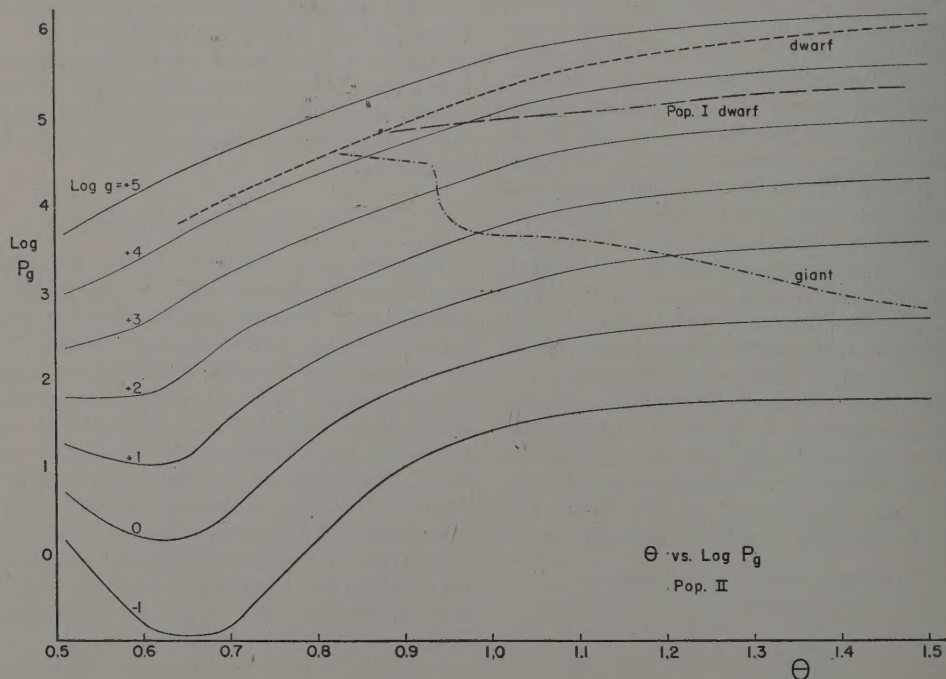


FIG. 2.

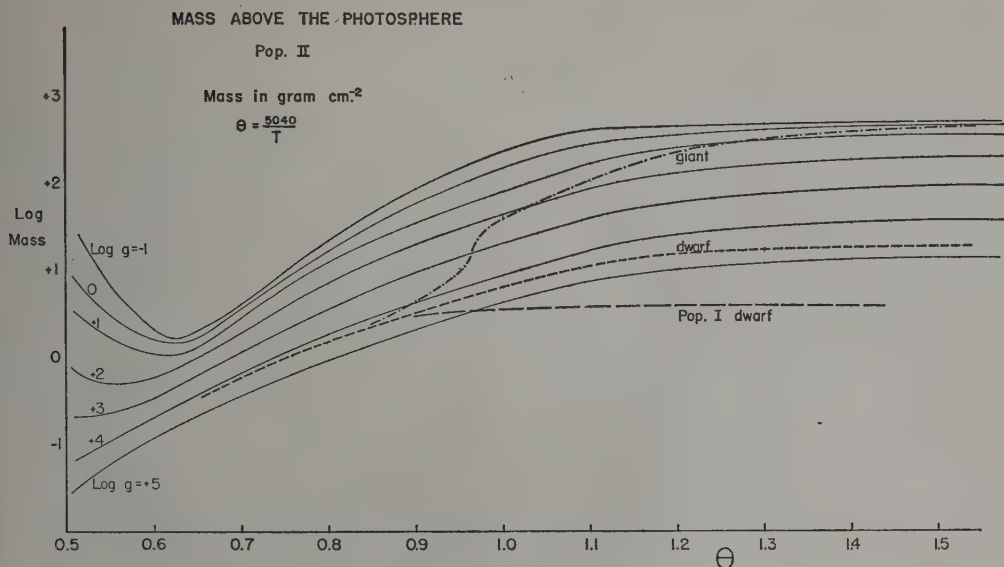


FIG. 3.

may make a considerable contribution to the "continuous" absorption. Further calculations (based on Vihart's new set of model atmospheres) are projected.

MEAN ATMOSPHERIC PARAMETERS

Pending the detailed calculations one may make some rough predictions concerning the properties of subdwarf atmospheres. I shall describe here the results of some calculations by our Michigan group in which we supposed that the hydrogen/metal ratio was 30 times greater than in the solar atmosphere, log depletion = 1.50; i.e., the ratio corresponded roughly to something between that found in HD 219617 and HD 19445. It was assumed that the spectrum was formed at an optical depth $\tau = 0.4$. First, the relation between gas and electron pressure was calculated for this metal-poor mixture (Aller 1953; 1960). Then the relation between gas pressure and absorption coefficient was found, i.e., $\kappa(P_g, T)$, for $\lambda 4250$. If κ is taken as equal to its mean value in the non-producing layers, then

$$P_g = (g/\kappa)\tau_0.$$

With τ_0 chosen as 0.4, we derived P_g as a function of $\theta = 5040/T$ with the surface gravity, g , as a parameter. Likewise, since $P_g(P_g, \theta)$ is known, we can find $P_e(\theta)$.

The surface gravity is known as a function of surface temperature for normal main-sequence stars. We assumed the same relation (Russell 1934) to hold for main-sequence stars of population Type II, viz.

$$\log g = 3.79 + 0.64 \theta$$

where g is given in cgs units). Calculations were also extended to the giant stars in the globular clusters. In accordance with present evolutionary concepts, the

mass was taken as a constant equal to 1.3 solar masses. The bolometric magnitudes and temperatures were taken from the work of Sandage (1954), and the corresponding surface gravities calculated in a straightforward way.

The relation between gas and electron pressure differs from that obtained for stars of normal composition in that hydrogen continues to supply electrons to much lower temperatures. In the subdwarfs, of temperature 6000°K or more, the electrons are all supplied by hydrogen. At temperatures as low as 4000°K the elec-

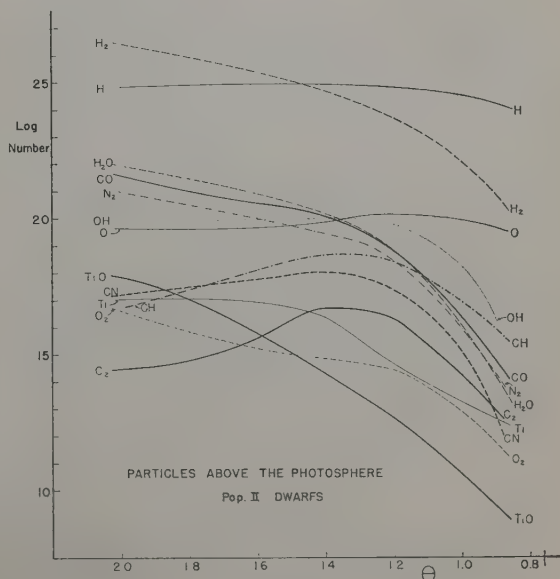


FIG. 4.

trons are supplied by the metals as in normal dwarfs. Hence, Greenstein has noted that it is not surprising that he has been unable to find weak-lined objects among the very coolest stars. Notice that at these lower temperatures in the subdwarfs, the opacity per gram is greatly reduced since the metals supply the electrons for the formation of the negative hydrogen ion. The total amount of material above the photosphere is thus greatly enhanced, but the number of metal atoms above each square centimeter of the surface is roughly the same as in population Type I stars.

Here I shall summarize the results only for the main-sequence stars which are tentatively identified with subdwarfs. At 6000°K the electron pressure appears to be about the same in these stars as in the sun, but at 5100°K, $\partial \log P_e$ is about 0.4 and increases to about 0.6 as the temperature drops to 4200°K; the electron pressure is higher in the normal dwarfs. An opposite behavior is shown for the gas pressure which is the same in subdwarfs and normal dwarfs at 5800°K, but increases at a faster rate in the subdwarfs as the temperature is lowered. At 4200°K the gas pressure is of the order of four times greater in a subdwarf than in a normal dwarf. Likewise the mass above the photosphere is greater in the subdwarfs than in dwarfs at low temperatures, the factor being of the order of four at 4200°K. Accordingly, we might expect more pronounced pressure broadening effects in cool subdwarfs than in cool normal dwarfs of the same temperature. We would expect subdwarfs to be cooler than normal stars of the same level of ionization although the effects would be small. The predicted behavior of the lines would be roughly similar in the two sequences.

MOLECULAR EQUILIBRIA AT LOW TEMPERATURES

The compounds formed in atmospheres of cool subdwarfs naturally emphasize the hydrides. We have employed the equilibrium "constants" published by Pecker and Peuchot (1934). The concentrations of the various compounds may be computed in a straightforward manner as has been described by many workers (e.g., Russell 1934). The calculations differ from those of deJager and Neven (1957) in that not only are different abundance ratios chosen, but an attempt is made (although crudely) to allow for the varying depth of the photosphere. Such calculations are necessarily very uncertain at the low temperatures where the sources of opacity have not all been identified. For a rough calculation we assume the following ratios:

Assumed $\log N(\text{element})/N(H)$				
C	N	O	Ti	Zr
-5.1	-5.4	-4.6	-8.8	-11.2

Probably carbon is actually less abundant than we have assumed. At present nothing is known about nitrogen and oxygen, although data on these atoms could be found by extending the calculation to the infrared. At the higher temperatures *OH* and *CH* are the most abundant of observable molecules; *CN* is very weak. As the temperature falls, *CO* and *H₂O* become the most abundant molecules after *H₂*. Below 3400°K, molecular hydrogen is the most abundant particle in the atmosphere; at 2800°K, *H₂* is ten times as abundant as atomic hydrogen. The calculation of the number of atoms above the photosphere at low temperatures becomes unrealistic because of neglected factors in the opacity. *CN* and *CO* have a maximum near 3700°K and then fall off to low temperatures as the carbon gets tied up in *CO*. Comparison with data secured for normal dwarfs indicates that in a general sort of way, the spectra should resemble those of normal dwarfs as far as molecules are concerned.

ACKNOWLEDGMENTS

Particular thanks are due to Miss Pyne and Miss Ramseyer who carried out the extensive numerical calculations of mean values of P_g , P_e , mass above the photosphere, and dissociation equilibria. The diagrams were drawn by Peter Boyce.

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A Series of Subdwarf Atmospheres*

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A SERIES of subdwarf atmospheres has been constructed on the Los Alamos IBM 704 calculators. The temperature range is 4200° to 8400°K, and the metal abundances are approximately 1/3, 1/10, and 1/100 that of the sun. The continuous spectra of the models compare satisfactorily with those observed in subdwarfs for wavelengths above about 4000 Å, but at shorter wavelengths there are important discrepancies. In particular, the absorption coefficient which was used would be more sensitive to the metal abundance.

Bolometric corrections were computed for the models, and this allowed a relation between absolute visual magnitude and radius to be determined. Although the uncertainties are too large to be able to form a definite conclusion, this M_v-R relation in connection with an observed H-R diagram indicates that subdwarfs have nearly the same radii and luminosities as normal stars of the same effective temperatures.

* Complete paper to be published in the *Astrophysical Journal* Supplement Series.

The transcription of the discussions following the last two papers was not sufficiently accurate for publication.

Southern Hemisphere Photometry. VIII. Cepheids in the Small Magellanic Cloud

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(Received June 30, 1960)

Photographic and photovisual data are presented for 69 cepheids in the Small Magellanic Cloud. The region investigated is centered on NGC 371 and has a projected area of about $1^\circ \times 1^\circ$. The new photoelectrically calibrated magnitude scales are compared to the previous Harvard magnitudes and corrections reaching $\frac{1}{2}$ magnitude at $B=17.5$ mag. are noted.

Light curves in B and V are presented for 64 of the cepheids. The light curves compare well in general shape wherever previous Harvard light curves are available, but they do not correspond well in secondary details, such as "bumps." Periods determined in the span 1955–1956 are given, together with their estimated uncertainties. An indication that some of these cepheids are shortening their periods is discussed.

Magnitudes at mean light intensity, $\langle B \rangle$ and $\langle V \rangle$, are obtained by a method of correcting magnitude means. $\langle B \rangle$, $\langle V \rangle$, and the light curve parameters of amplitude, asymmetry, and rate-of-rise are given for the cepheids. The average error in $\langle B \rangle$, $\langle V \rangle$ magnitudes for the bright cepheids is estimated to be ± 0.03 mag., increasing to about ± 0.05 mag. for the fainter cepheids.

The residuals from the mean period-apparent magnitude relation, $\Delta \langle B \rangle$, and the residuals from the mean color-magnitude relation, $\Delta(\langle B \rangle - \langle V \rangle)$, are tabulated. Cepheids which are intrinsically redder than average are expected to also be intrinsically fainter than average. In fact, the intrinsic ratio of $\Delta \langle B \rangle^0$ to $\Delta(\langle B \rangle - \langle V \rangle)^0$ is so close to the reddening-to-absorption ratio that the close correlation which exists between these residuals cannot be used to determine the reddening for the cepheids.

Instead, a method is developed whereby only $\langle B \rangle$ and the amplitude, $B_{\min} - B_{\max}$, are needed to obtain absorption for individual cepheids. To accomplish this it is shown that the residuals from the mean period-amplitude, period-asymmetry and period-rate-of-rise relation is large compared to observational errors. Therefore, the shape of the light curve as indicated by these parameters is indicative of where a cepheid falls in the instability gap in the color-magnitude diagram. Because the absorption in the particular SMC field investigated is not large, the intrinsic relation between $\Delta \langle B \rangle$ and $\Delta(B_{\min} - B_{\max})$ is not scattered be-

yond recognition. Deviations from the inferred narrow, intrinsic relation can be measured and attributed to absorption. The mean reddening in the field determined in this way is $\langle E_{B-V} \rangle = 0.05$ mag. This value checks very well with the mean reddening determined from three-color measures of supergiants in this field. Because this method of determining reddening only requires B light curves, and because the absorption measures are 4 times the reddening, the method seems promising for use in faint galaxies where accurate color measures are difficult.

All the cepheid parameters are then corrected individually for reddening and absorption, and tabulated. The unreddened period-color and color-magnitude relations are shown. The $P-L$ relations are:

$$\begin{aligned} \langle B \rangle^0 &= 17.45 - 2.25 \log P \\ \langle V \rangle^0 &= 17.05 - 2.47 \log P \end{aligned} \quad \left. \begin{array}{l} \text{eye estimated linear fit to unreddened} \\ \text{magnitudes at mean light.} \end{array} \right\}$$

From the position of the unreddened points in the color-magnitude diagram, the slope of the lines of constant period is empirically determined. On the assumption that the $P\sqrt{\rho} = \text{const.}$ law is applicable to cepheids, the $\log T_e$, $(\langle B \rangle - \langle V \rangle)^0$ relation which is needed to predict the observed slope of the constant period lines is computed. The effect of nonhorizontal evolution through the cepheid gap, indicated by young SMC star clusters is included in the computation. The empirically determined slope of the lines of constant period is confirmed by its prediction of the observed dispersion about the $P-L$ relation and its prediction of the observed slope of the $\Delta(\langle B \rangle - \langle V \rangle)^0$, $\Delta \langle B \rangle^0$ relation.

One cepheid in the present sample is like the cepheids with periods around one day and shorter, and at about $B=17$ mag which Detarayet and Landi Dessy showed to be members of the SMC. The results on the cepheid observed here, $P=1^d$, suggest that this group is probably varying in higher modes and bears the same relation to the longer period SMC cepheids as the RR Lyrae c types do to the RR Lyrae a and b types.

It is pointed out that cepheids appear to evolve through the gap in the SMC clusters along a line of constant period. This circumstance may be accidental, but, if it is not, there seems to be no explanation for why it is so.

CONTENTS

Observations

1. Introduction.
2. Observing Techniques.
3. Standard Magnitude Sequence.
4. Supplementary Standards Near Edge of Region.
5. Selecting the Cepheids and the Area Observed.
6. Comparison to Harvard Magnitude Scale and Harvard $P-L$ Relation.
7. Measuring the Cepheids.
8. The Periods.
9. The Light Curves.
10. The Light-Curve Parameters.
11. Crowding, Doubling, and Uncertain Periods.

12. Accuracy of Parameters and Sources of Error.
13. The Measured $P-L$ Relation.
14. Observed Place of the Cepheids in the Color-Magnitude Diagram.

Analysis of the Data

15. The Period-Amplitude Relation.
16. The Period-Asymmetry Relation.
17. The Period-Rate-of-Rise Relation.
18. Deriving the Reddening of the Cepheids.
19. The Unreddened Cepheid Magnitudes and Colors
 - (a) The Period-Color Relation.
 - (b) The Period-Luminosity Relation.
 - (c) The Color-Magnitude Relation.
20. The Lines of Constant Period in the Color-Magnitude Diagram.
 - (a) Nonhorizontal Evolution.

* Publication of the Goethe Link Observatory of Indiana University No. 38.

6. The Dispersion About the Mean P—L Relation.
7. The Correlation Between Deviations from the Mean P—L Relation and Deviations from the Mean Color-Magnitude Relation.
8. The Width of the P—L Relation and Color-Magnitude Domain.
9. SMC Cepheids with Periods Near One Day and a Test for Overtones.
10. Nonvariable Stars in the Same Field as the Cepheids.
11. Evolution of Cepheids in the Color-Magnitude Diagram.
12. Period Changes.

OBSERVATIONS

1. Introduction

PAPERS III, V, VI, and VIII of this series (Arp 1958 c, d; 1959 a, b) presented color-magnitude diagrams of stars in various clusters and fields within the SMC. Originally the major aim of that photometric investigation was to derive very accurately the distance modulus to the SMC by means of these color-magnitude diagrams. The intention was then to use this modulus in conjunction with special kinds of objects, such as cepheids, to calibrate distance criteria or use in more remote galaxies.

Since it turned out that the color-magnitude diagrams obtained in the SMC did not resemble very closely diagrams of any stars in our own galaxy, there is still uncertainty as to the precise modulus of the SMC. Since there is a range in ages of the stars in both systems, presumably the cause of the differences in the color-magnitude diagrams is different chemical composition between the stars in our system and in the SMC. Therefore, even if we knew the precise modulus to the SMC, the cepheids could not be expected to be identical to cepheids in our own or other galaxies. Indeed, the observations which are presented in the following pages show that the SMC cepheids are different in several respects.

The effect of a change in chemical composition on the period-luminosity law is at present not known. Therefore, the distance modulus to the SMC from this criterion and the absolute magnitudes of the cepheids in this and other systems is still somewhat uncertain. It is clear that before we can calibrate more accurately the cepheid luminosities we need to know more about the pulsation process itself. Fortunately the data presented in the present paper appear to be accurate enough to give a much better understanding of how cepheids behave as a function of temperature, luminosity, and period. Once this, or other data, enable the effect of chemical change on the period-luminosity law to be calculated, then the modulus to the SMC can be obtained from the present data. Of course, the luminosity of cepheids may turn out to be insensitive to such things as composition differences and then the P—L relation presented here could be used straight-

away in all systems to derive distances. Nevertheless, at the moment we cannot demonstrate this to be true, and therefore do not know the size of the error we might make in assuming it to be true.

2. Observing Techniques

The Royal Observatory at Capetown kindly made their McClean refracting telescope available on a number of nights each month between November, 1955, and July, 1956. That twin refractor was used to expose two photographic plates simultaneously. An Eastman 103a-O plate behind an Omag 301 yellow filter was exposed through the visually corrected 18-in. diameter lens. An Eastman 103a-O plate behind a GG13 ultraviolet cutoff filter was exposed through the 24-in. photographically corrected lens. This procedure enabled sets of photographic plates to be obtained which (1) Yield magnitudes on the Johnson-Morgan B, V system when measured relative to B, V standards on the same plate; (2) Have limiting magnitudes fainter than $B=18$ and $V=17$ for one-hour exposures with average seeing; (3) Record photographic images at a scale of $30''.64/\text{mm}$ over an area more than 1° square which is free from optical aberrations.

A total of 27 plates in each color were obtained in this way for the cepheid field investigated here. The

TABLE I. Standard magnitude sequence for measurement of cepheids.

Star	V p.e.	B values	V adopted	B values	
Primary Standards					
a	11.09	12.27	11.05	12.27	star Z in SHP II
b	13.91	15.48	13.86	15.44	
c	13.35	13.82	13.34	13.80	
d	16.61	16.31	16.56	16.38	
e	15.91	17.26	15.93	17.25	
Supplementary Standards					
f			14.43	14.56	
h			16.46	16.80	
i			16.15	16.01	
j			16.17	17.07	
k			15.95	15.95	
2064a			11.75	12.64	
b			14.26	14.21	
c			15.38	15.29	
843a			12.64	13.80	
b			13.91	14.42	
c			14.94	15.03	
d			15.03	15.53	
e			15.99	16.12	
837a			14.08	14.09	
b			14.85	15.01	
1898a			14.08	14.05	
b			14.48	14.62	
c			15.95	16.13	
d			14.97	16.32	
e			16.58	16.73	
856a			14.78	14.79	

supplementary standard at a sky difference of zero. This procedure was used to derive the standard magnitudes of all the standards in Table I which are prefixed by an HV number. Figure 1 is only a sample of three stars showing, as expected, how the sky background difference becomes more important as the star becomes fainter. The procedure is of course somewhat approximate because the two effects should not exactly cancel each other out. It is the only way to proceed under the circumstances, however, and checks were available from photoelectric sequences in various outer regions discussed in Sec. 3.

5. Selecting the Cepheids and the Area Observed

The area investigated here for cepheids is shown in plate I. It is centered on the loose open cluster NGC 371. This particular area was given priority because (1) It contained the maximum number of cepheids without serious crowding by the star field to this magnitude level; (2) It has the minimum variation of sky background light, and therefore presumably the least obscuration of the available regions. (3) It covered a region containing very bright young stars as well as regions which appeared to be representative of the general background of the SMC; (4) It was near regions which had been investigated to faint limits in earlier papers in this series.

A list which summarizes all cepheids which have

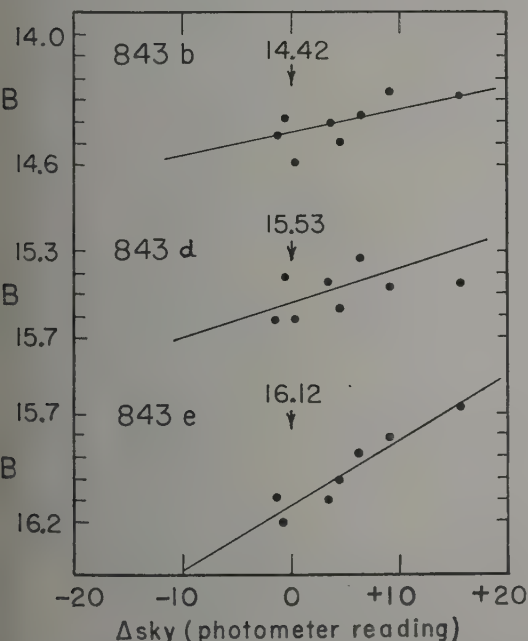


FIG. 1. Dependence of derived magnitude on value of sky background for supplementary standards in outer regions of field. Each point represents the magnitude derived, plotted against the measured difference in sky background between the sky near the supplementary standard and the sky near the primary standard which it is referred. The adopted magnitude is read at $\Delta \text{sky} = 0$.

TABLE II. Magnitude measures in Harvard magnitude sequences.

Harvard sequence no.	<i>V</i>	Arp <i>B</i> *	Harvard <i>m</i> ₂₉
15 <i>s</i>	13.41	13.35	13.13:
<i>t</i>	13.97	13.71	13.73
<i>u</i>	14.32	14.22	14.11
<i>v</i>	14.70	14.97	14.74
<i>w</i>	15.46	15.34	15.21
<i>x</i>	15.63	15.63	15.35
<i>y</i>	14.77	16.10	15.63
<i>z</i>	(16.39)	16.46	15.96
<i>α</i>	(16.99)	16.96	16.41
42			
<i>v</i>	14.61	14.73	14.39
<i>w</i>	14.52	14.66	14.59
<i>x'</i>	14.09	15.32	15.06 double
<i>α'</i>	15.64	16.19	15.66
<i>β</i>	15.98	16.28	15.85
<i>γ</i>	(16.88)	16.64	16.34
<i>δ</i>	(16.72)	17.42	16.71 double

* Mean of measures on two plates for stars in Sequence 15 in *B*; rest of measures from single plate.

Harvard variable numbers in the SMC has been prepared at Mt. Stromlo. This list, kindly sent to me by Gascoigne, enabled the positions of 180 cepheids to be plotted over a region of $76' \times 76'$ centered on NGC 371. Coincidence of these plotted positions with stars on the photograph of the right approximate brightness enabled positive identification of the majority of the cepheids. Nevertheless, a number of photographic plates were blinked to check that all the cepheids identified were stars which were actually variable on the plates to be measured. In this process a number of uncertain identifications were resolved and two new variables were discovered (Arp a and Arp b).

All single uncrowded cepheids within $25'$ of the center of the plate were selected for measurement. In order to obtain the largest possible sample of the rare long-period cepheids, all cepheids of period greater than 10 days were measured regardless of how far from the plate center they fell. In this way a group of 69 cepheids were selected for measurement. This was the largest statistical sample that it was felt feasible to undertake to measure. The sample included as many long-period cepheids as possible and restricts measurement to the central portion of the plate, particularly for the faint variables, as much as possible.

6. Comparison to Harvard Magnitude Scale and Harvard P—L Relation

The best-known and most often used cepheid P—L relation is that published by Shapely (1940). The magnitude sequences used in that determination are given in another publication by McKibben (1940). Two of those Harvard magnitude sequences, Nos. 15 and 42, are centrally located in the region investigated here. Table II lists the magnitudes of some stars in these Harvard sequences as measured on two refractor *B* plates for sequence No. 15 and one *B* plate for No. 42.

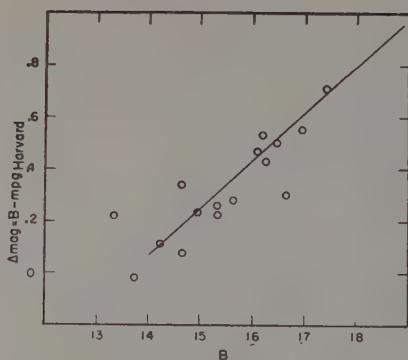


FIG. 2. Corrections to the original Harvard SMC magnitudes (new magnitudes minus old) as a function of the new magnitudes, B .

V magnitudes for these stars, as measured on one plate, are also given in Table II.

There is a systematic scale error in the Harvard magnitude sequences relative to the magnitude scale based on photoelectric measures which are used here. This systematic difference in the magnitudes is illustrated in Fig. 2, where the differences, from Table II, between the Harvard magnitudes and the B magnitudes are plotted as a function of B .

A similar scale difference is indicated by comparison of Harvard light curves to light curves published by Gascoigne (1954). It would seem that it is the primary Harvard sequence I in the SMC which is in error. The details of the origin of that sequence are difficult to discover, but apparently magnitudes were photographically transferred from the north polar sequence to the C regions, C1 and C2. Then a combination of photographic transfers and diffraction grating techniques were used to transfer to the sequences into the SMC (Shapley 1924). The divergence between the photoelectrically based magnitude scales in the SMC and the original photographic sequence is very similar to the divergences of the photographically transferred Selected Area magnitudes which were discovered by Stebbins, Whitford, and Johnson (1950), and may indicate that the major error in the Harvard SMC magnitude scale originated in the C-region magnitude scale.

To check the consistency of our results so far, we can approximate Fig. 2 by the following straight-line relationship:

$$\bar{m} = 0.815B + 2.52 \quad 14 < B < 18, \quad (1)$$

where $\bar{m}$ is the mean Harvard photographic magnitude of the cepheids which gave Shapley the following P-L relation:

$$\bar{m} = 17.04 - 1.74 \log P,$$

Shapley: original magnitudes; (2)

by substituting (1) in the above equation to correct

the magnitude scale, we obtain:

$$B = 17.82 - 2.14 \log P,$$

Shapley: corrected magnitudes, (3)

which is very close to the P-L relation derived in the present paper:

$$\langle B \rangle = 17.7 - 2.2 \log P, \text{ Arp: present investigation.}$$

It is interesting to note that the P-L relation originally derived in the Large Magellanic Cloud (Shapley 1940) was closer to the new P-L relation for the SMC. The original magnitude scale in the LMC may have been more accurate than that in the SMC, although this was not the feeling of Mohr (1937). Whether there is any intrinsic difference in the P-L relations between the LMC and SMC must await completion of a current investigation of the LMC by Eggen, Sandage, and Wooley (unpublished), and Hodge (unpublished).

It should be noted that there is a zero point difference between B and the old photographic scale. By definition the B magnitudes (Johnson and Morgan 1953) are about 0.17 mag. fainter than m_{pg} (for stars of average color, see Arp 1957). Of course, brighter than $B = 14$ mag., the original Harvard scale is quite good, attested to by many observers (Gascoigne 1954). Fainter than $B = 14$, however, scale errors are present in the early SMC measures, and, as shown by Stebbins, Whitford, and Johnson (1950), in early extragalactic measures in general. These scale errors undoubtedly account for such effects as the apparent nonlinearity of the P-L relation determined from extragalactic nebulae as discussed by Kukarkin (1937), Ledoux and Walraven (1958, p. 377).

7. Measuring the Cepheids

All the plates on which the magnitudes of the cepheids were measured are listed in Table III. The prefixed letter M designates the McClean refractor. The plate number follows the M, and in the immediately adjoining column the Julian date of the midpoint of the exposure is given. These times have not been given a heliocentric correction.

The individual measures on 69 cepheids follow in successive columns in Table III. The Harvard variable number is at the head of each column and the cepheids are arranged in order of period with the longest period first, except for the last five. The periods of these last five cepheids could not be determined satisfactorily and they were added to the end of Table III only in order to complete the record.

Since the B and V plates were always taken together the time of mid-exposure is the same for both. Only the case of M 81 and M 82 are they different. The difference was caused by allowing the V exposure to run on longer than the B . There is a companion plate for every V plate except for M 26 where the cor-

TABLE III

MEASURES OF CEPHEIDS

Plates	Dates, J. D.		HV 1877	HV 837	HV 11182	HV 2064	HV 840	HV 1967	HV 847	HV 1884	HV 1925
	2,435, 426, 569+		49.5 ± .3	42.6 ± .4	39.6 ± .2	33.7 ± .3	33.1 ± .2	29.1 ± .1	27.2 ± .1	18.11 ± .04	17.18 ± .07
M3, M4	0	0	14.32	..	14.70	..	14.18 13.33	14.54 13.49	13.80	..	14.32 13.67
M7, M8	1.007	1.007	14.31 13.08	13.45 12.85	15.00 13.79	(13.92) 13.19	14.66 13.53	13.86 13.19	15.26 14.04	15.72 14.51	14.20 13.73
M11, M12	2.914	2.914	14.37 13.30	13.63 12.91	15.02 14.10	(14.01) 13.35	14.70 13.63	14.06 13.34	15.28 14.16	15.46 14.52	14.39 13.90
M21, M22	4.828	4.828	(14.20) 13.43	..	..	..	..	..	..	..	..
M26	..	..	..	..	..	..	..	..	..	..	..
M31, M32	22.802	22.802	13.48 12.98	14.40 13.29	14.37 13.56	15.12 14.00	13.80 12.95	14.02 13.45	14.73 13.87	15.27 14.50	14.72 14.00
M37, M38	24.910	24.910	13.40 (12.67)	14.54 13.41	14.50 13.62	15.13 13.81	13.92 13.10	13.69 13.00	14.89 13.93	14.55 13.03	14.86 14.00
M41, M42	25.990	25.990	13.40 11.88	14.54 13.48	14.65 13.55	15.17 14.08	14.10 13.20	13.62 13.29	15.08 13.91	14.68 13.92	14.86 14.06
M49	29.841	..	..	..	..	..	..	..	..	..	..
M55, M56	30.991	30.991	13.50 (12.78)	14.80 13.50	14.93 13.50	14.92 14.09	14.39 13.39	13.87 13.23	15.13 14.11	15.19 14.17	14.01 13.36
M61, M62	31.952	31.952	13.46 12.72	14.72 13.52	14.90 13.70	14.81 13.90	14.51 13.41	14.10 13.41	15.22 14.16	15.37 14.22	13.85 13.54
M69, M70	49.832	49.832	..	..	..	..	..	..	..	..	..
M73, M74	54.837	54.837	14.22 (13.33)	13.99	14.19	..	14.96 (13.94)	13.87	..	15.21 (14.07)	15.69 (14.68)
M81, M82	77.747	77.750	13.57 12.82	14.20 13.29	14.82 13.68	14.36 13.42	15.00 13.97	(14.79) 13.88	14.90 13.88	15.20 14.31	14.93 13.96
M89, M90	79.780	79.780	13.59 12.69	13.59 12.87	15.13 13.79	14.47 13.60	15.14 14.12	14.44 13.77	15.08 13.85	14.52 13.81	14.46 14.08
M93, M94	82.796	82.796	13.61 12.74	13.30 12.67	14.93 13.73	(14.64) 13.57	14.09 13.49	13.66 13.09	15.23 14.09	14.94 14.10	13.99 13.40
M101, M102	84.823	84.823	13.71	..	..	..	..	..	..	..	..
M107, M108	90.796	90.796	13.96 13.07	13.72 12.98	13.91 13.30	15.10 13.95	13.83 13.22	14.11 13.35	14.67 14.01	15.72 14.64	14.56 13.80
M113, M114	91.797	91.797	14.16 13.10	13.73 12.93	14.19 13.31	15.18 14.08	14.00 13.10	(14.23) 13.35	14.29 13.56	15.63 14.52	14.75 13.79
M117, M118	92.809	92.809	14.18 13.05	13.83 12.87	(13.87) 13.30	15.29 14.09	14.10 13.16	14.24 13.37	13.92 13.45	15.51 14.48	14.90 14.05
M137, M138	177.080	177.080	13.60 (12.78)	..	..	..	..	..	..	..	..
M149, M150	200.990	200.990	14.36 (13.19)	14.67	..	..	..	..	..	..	..
M151, M152	208.049	208.049	14.27 (13.69)	13.70 (13.07)	14.09	..	..	..	..	..	..
M159, M160	209.025	209.025	14.40 13.39	13.61 (12.71)	13.89 13.22	14.25 13.39	15.06 13.87	14.33 13.52	14.52 13.70	14.89 14.10	14.45 13.90
M163, M164	209.947	209.947	14.51 13.33	13.23 (12.71)	14.11 13.27	14.19 13.30	15.14 13.94	14.31 13.51	14.70 13.70	14.98 14.11	14.61 13.75
M171, M172	229.904	229.904	13.62 12.89	14.20	..	..	..	..	..	..	..
M179, M180	230.925	230.925	13.68 12.72	14.36 13.12	15.01 13.67	15.19 14.13	(14.61) 13.40	13.80 13.12	14.17 13.43	15.29 14.18	14.89 13.90
M183, M184	231.910	231.910	13.66 12.87	14.38 13.19	15.02 13.68	15.16 14.04	(14.57) 13.49	13.80 13.09	14.21 13.57	15.34 14.33	14.88 14.03

TABLE III—Continued

Plates	HV 1954	HV 1787	HV 843	HV 1695	HV 1933	HV 1873	HV 1744	HV 856	HV 1707	HV 2083
	16.71 ± .02	16.22 ± .04	14.70 ± .02	14.50 ± .05	13.76 ± .04	12.91 ± .02	12.59 ± .03	12.13 ± .03	11.43 ± .02	11.18 ± .02
M3, M4	14.96 13.98	14.87	16.57 15.37	..	14.76 14.22	15.53	14.92	15.01 14.61	15.13 14.47	15.59 14.81
M7, M8	15.00 14.04	15.22 14.03	16.52 15.18	15.11 14.38	14.56 13.98	15.31 14.75	14.70 14.13	14.82 14.52	15.38 14.49	15.59 14.72
M11, M12	14.70 13.93	15.54 14.42	..	14.95 15.31	14.78 13.92	14.88	14.35 14.06	15.00 (14.48)	15.63 14.61	15.90 14.12
M21, M22	14.47 13.88	..	14.40	..	14.50	..	14.59	..	14.72	..
M26	..	..	14.61	..	14.79	..	14.79	..	14.76	..
M31, M32	14.26 13.69	15.48 14.66	15.27 14.56	..	14.71	15.12 14.49	15.91 15.04	15.40 14.76	15.75 14.84	15.20 14.50
M37, M38	14.73 13.32	15.01 14.26	15.72 14.89	15.25	14.40	15.77 14.98	14.90 14.28	15.40 14.45	15.40 14.45	14.98 14.14
M41, M42	13.99 13.46	15.19 14.26	15.98 14.70	14.65 14.13	15.12 14.49	15.50 14.86	14.84 14.19	15.10 14.30	15.50 14.70	15.01 14.33
M49	(14.59)	..	..	..	..	..	..	..	15.27	15.68
M55, M56	14.62 13.93	14.69 13.85	16.03 15.00	15.47 14.27	14.23 13.85	15.14 14.63	15.01 14.27	..	15.24 14.40	15.74 14.80
M61, M62	14.83 13.99	14.83 14.06	15.62 14.71	15.68 14.38	14.75 (14.20)	15.24 14.60	15.34 14.40	15.82 14.85	14.86 14.30	15.92
M69, M70	14.91 14.01	..	14.29	(15.01) 14.33	15.73 14.65	(15.17) 14.45	15.71 14.99	15.02 14.39	14.84 14.30	15.93 14.84
M73, M74	14.54 13.87	15.68 (14.63)	15.78 14.83	14.70	14.89	14.86 14.31	15.00 14.19	(15.51) (14.71)	14.77 14.30	15.91
M81, M82	14.26 13.71	14.40 13.82	15.67 14.83	15.31 14.63	15.20 14.31	15.71 14.78	14.21 13.84	15.40 14.79	14.69 14.02	15.81 14.69
M89, M90	14.80 13.82	14.77 13.99	15.06 14.39	15.75 14.65	15.26 14.43	14.80 14.20	14.81 14.16	15.59 14.77	15.10 14.50	15.29 14.70
M93, M94	14.85 14.00	15.21	15.54	15.14	14.56	14.74 14.04	15.24 14.50	15.91 14.81	15.70 14.54	..
M101, M102	14.91 14.07	(15.55) (14.34)	16.18	..	14.14	(14.12) (13.70)	(15.38)	..	15.17 14.80	15.86 14.68
M107, M108	13.89 13.36	15.16 14.30	15.91 14.88	15.26 14.50	15.17 14.39	15.57 14.70	14.16 13.85	15.61 14.78	14.99 14.27	15.42 14.77
M113, M114	13.72 13.46	15.45 13.82	15.86 14.97	15.36 14.49	..	14.39	15.32 14.43	14.68 14.14	15.62 15.09	15.20 14.40
M117, M118	14.06 13.48	14.18 13.80	15.41 14.75	15.39 14.58	15.27 14.34	14.92 14.26	14.82 14.22	15.62 14.91	15.32 14.59	14.91
M137, M138	..	13.59 14.79 14.00	16.45 15.21	15.37 14.29	15.14 14.33	15.70 14.80	14.70 14.21	15.80 14.90	15.38 14.70	15.81 14.98
M149, M150	15.02	..	..	..	15.40 14.80	15.24	..	..	15.26 14.52	15.69
M151, M152	13.76	..	..	..	15.53 14.77	15.62	..	..	15.50	..
M159, M160	13.73 13.40	(14.72) 14.11	15.89 14.97	15.50 14.63	(14.18) 13.81	14.71 14.27	15.40 14.49	15.19 14.51	15.84 14.64	15.80 14.72
M163, M164	14.03 13.51	14.66 14.10	15.80 14.97	15.61 14.72	14.31 13.88	14.91 14.39	15.42 14.64	15.14 14.54	15.78 14.75	15.87 14.75
M171, M172	14.48 13.79	15.36 14.38	15.57 14.56	14.73 14.10	15.30 14.49	15.83 14.91	14.37 13.89	15.60	15.31	15.38
M179, M180	14.42 13.82	15.40 14.30	(15.70) 14.78	14.84 14.40	15.32 14.46	14.80 14.07	14.80 14.07	14.95 14.50	15.49 14.73	15.69 14.79
M183, M184	14.66 13.90	15.55 14.38	(15.91) 14.75	14.91 14.31	15.14 14.40	15.70 14.98	15.07 14.34	..	14.28	15.53 14.63

TABLE III—Continued

Plates	HV 2060 10.21 ± .01	HV 1768 9.798 ± .01	HV 1950 7.980 ± .01	HV 1905 7.413 ± .01	HV 1855 6.830 ± .006	HV 1945 6.465 ± .005	HV 11193 6.440 ± .008	HV 1858 6.120 ± .007	HV 1892 5.660 ± .005	HV 1929 5.579 ± .007	
M3, M4	15.33 14.60	15.54	16.02 15.16	15.25 14.68	15.82	16.20 15.45		16.13	16.22 15.31	16.09 15.50	
M7, M8	15.29 14.61	15.82 14.92	15.71 14.91	15.63 15.00	16.08 15.18	16.40 15.61	16.54 15.66	16.20 15.69	15.98 15.30	16.17 15.55	
M11, M12	14.98 14.61	15.92 15.15	15.41 14.76	15.95 15.17	16.47 (15.70)	15.34 14.88	16.68 16.01	16.35 15.98	16.38 15.49	15.76 15.2	
M21, M22	..	13.83	..	14.88	15.8	..	14.95	..	15.55	..	
M26	..	14.00	..	14.89	..	15.11	..	15.30	..	15.74	
M31, M32	15.12 (14.13)	15.92 15.18	16.13 15.01	15.31 15.0	16.30 15.59	15.39	16.45 15.67	..	15.31	16.20 15.49	
M37, M38	(14.50)	15.42 14.89	15.75 14.73	15.70 15.14	15.88 15.14	15.88 14.94	16.11 15.37	16.30 15.51	16.38 15.33	15.74 15.06	
M41, M42	14.52 14.03	15.48 14.65	14.53 14.63	15.77 15.35	15.58 14.90	16.20 15.51	16.52 15.69	16.42 15.60	16.24 15.48	15.99 15.54	
M49	..	..	..	..	16.36	..	..	..	16.00	..	
M55, M56	..	(14.60)	15.75	..	16.29	..	15.80 14.90	16.19	..	..	
M61, M62	(15.30) 14.75	15.89 15.00	15.94	..	15.11	15.69	16.02 15.40	16.32	16.32 15.60	16.43 15.50	
M69, M70	..	14.40	15.70	..	15.2	14.67	..	15.75 14.98	16.07 (15.29)	16.30	..
M73, M74	14.63	..	15.49 14.84	16.15	15.79	..	15.39	16.52	16.22	16.60	
M81, M82	14.67 14.19	15.50 14.82	15.84 14.91	15.92 15.23	16.44 15.45	16.27 15.40	16.36 15.57	15.81 15.12	16.60 15.73	16.09 15.45	
M89, M90	15.04 14.31	..	15.1	14.42	15.76	16.30	16.30	16.30	15.88	16.00 (15.28)	
M93, M94	15.34 14.65	15.74 14.92	15.47 14.57	15.62 14.81	15.88 15.14	15.85 15.01	16.22 15.16	16.04 15.49	16.30 15.72	16.17 15.13	
M101, M102	14.87	..	15.50 14.64	..	(15.89) (15.14)	16.41	..	16.80	16.00 (15.60)	..	
M107, M108	15.26 14.50	15.88 15.01	15.42 14.68	15.61 15.17	16.29 15.33	16.28 15.37	16.60 15.52	15.94 15.40	16.00 15.40	16.11 15.32	
M113, M114	15.33 14.63	15.82 15.12	15.57 14.63	15.87 15.19	16.40 15.39	16.40 15.36	16.82 15.70	16.20 15.48	16.09 15.30	15.58 15.01	
M117, M118	15.47 14.82	15.58 15.00	16.70 14.74	15.90 15.29	15.69 15.32	16.01 15.30	16.57 16.08	16.41 15.87	16.16 15.35	15.82 15.19	
M137, M138	14.54 14.08	15.87 14.82	15.54 14.80	15.38 14.78	15.78 14.97	15.81 15.10	16.42 15.60	16.23 15.29	16.10 15.26	16.00 15.36	
M149, M150	14.80	..	15.61	..	15.68	..	16.25	..	..	..	
M151, M152	14.64	..	15.80	15.81	15.15	16.39	16.48 15.55	16.61	16.25	16.55 15.65	
M159, M160	14.38	16.85 15.11	15.68 14.68	(15.59) 15.10	16.10 15.30	16.24 15.39	16.43 15.75	16.33 15.55	16.30 15.70	15.78 15.40	
M163, M164	14.63 14.13	15.73 15.05	16.38 14.49	15.81 (15.19)	15.60 14.91	15.30 14.80	16.3 15.85	16.59 15.69	15.84 15.28	15.91	
M171, M172	14.59 14.01	15.62 15.00	16.09 14.94	15.29 14.9	15.88 15.13	15.50 14.80	16.20 15.22	16.05 15.28	16.49 15.57	15.35 15.59	
M179, M180	14.70 14.18	15.42 14.70	16.10 14.97	15.56 14.88	15.59 14.80	15.78 14.91	16.19 15.24	15.78 15.12	16.55 15.72	15.83 15.08	
M183, M184	14.46 14.28	15.40 14.57	15.84 14.92	15.62 15.11	15.77 14.87	15.81 14.97	16.26 14.59	16.03 15.29	16.20 15.39	15.62 15.04	

TABLE III-Continued

Plates	HV 1903 5.091 ± .004	HV 1827 4.924 ± .002	HV 1934 4.885 ± .003	HV 1785 4.725 ± .003	HV 1966 4.270 ± .002	HV 1825 4.265 ± .003	HV 1994 4.217 ± .003	HV 1793 4.180 ± .004	HV 1891 3.451 ± .002	HV 11206 3.400 ± .002
M3, M4	16.80 . . .	15.80 . . .	15.97 15.37	15.76 . . .	15.75 15.31	16.56 . . .	15.71 15.25	16.29 . . .	16.10 15.33	16.61 15.9
M7, M8	15.80 . . .	15.72 15.06	16.32 15.59	16.47 15.50	16.21 15.58	15.83 15.18	16.23 15.72	16.63 16.01	15.85 15.90	16.92 16.20
M11, M12	16.28 15.49	16.20 15.54	16.61 15.70	16.84 16.0	15.24 14.74	16.50 15.19	16.86 16.11	16.40 15.83	15.78 15.42	16.22 15.59
M21, M22	. . . 15.83	16.1 15.35	. . . 15.33	. . . 15.23	(16.19) 15.47	. . . 15.37	16.19 15.62	. . . 15.62	. . . 15.85	. . . 16.2
. . . M26	. . . 15.57	. . . 15.14	. . . 15.70	. . . 15.80	. . . 15.63	. . . 15.39	. . . 15.81	. . . 16.13	. . . (16.45)	. . . 15.58
M31, M32	16.17 15.42	16.23 15.62	16.69 15.83	16.82 16.0	16.38 15.62	15.81 15.36	16.62 15.72	16.99 16.18	17.03 16.29	16.19 15.71
M37, M38	16.63 . . .	15.29 15.13	16.16 15.49	16.37 15.57	15.41 15.05	16.65 15.74	16.27 15.84	16.28 15.80	16.89 15.78	17.00 . . .
M41, M42	16.53 15.70	15.67 15.23	16.50 15.66	16.8 15.71	15.92 15.38	16.51 15.72	. . . 15.51	16.85 15.78	17.13 16.13	16.18 15.61
M49 . . .			16.21 . . .		(15.69) . . .			16.59 . . .		16.22 . . .
M55, M56		. . . 15.30		16.65 . . .	(16.33) . . .	15.68 . . .			15.92 . . .	16.71 . . .
M61, M62	15.59 15.25	16.21 . . .	16.68 . . .		16.48 . . .	15.94 . . .	16.90 . . .	16.43 15.95	16.52 (16.01)	17.22 . . .
M69, M70	16.35 . . .	15.40 15.18	16.25 15.60	16.6 . . .	(15.44) (15.03)	16.12 . . .	16.92 16.25	15.89 15.60	(17.18) > 16.2	16.26 . . .
M73, M74	16.32 . . .	15.38 15.10	16.29 . . .	16.7 . . .	15.43 . . .	16.31 . . .	15.78 . . .	16.32 . . .	15.72 . . .	16.72 . . .
M81, M82	15.73 15.09	16.51 15.18	15.96 15.30	16.45 15.59	16.19 15.52	15.51 15.22	16.72 15.74	16.07 . . .	16.35 . . .	16.48 15.7
M89, M90		15.72 . . .		16.70 . . .	15.28 (14.70)		(16.57) . . .			
M93, M94	15.64 15.27	16.34 15.65	. . . 15.30	16.61 15.8	16.29 15.70	15.89 15.44	16.54 (15.88)	15.81 15.30	16.13 15.30	. . . 15.95
M101, M102		15.85 . . .	16.45 . . .	16.88 . . .	(15.49) . . .	16.32 . . .		. . . 15.70		. . . 15.90
M107, M108	16.60 15.80	15.97 15.36	16.73 15.88	16.28 15.44	16.22 15.73	15.66 15.25	16.68 15.88	16.14 15.57	16.92 15.80	16.11 15.70
M113, M114		16.32 15.48	16.69 15.3	16.69 15.78	16.51 15.70	16.19 (15.49)	17.01 16.07	16.30 15.79	17.22 . . .	16.50 16.11
M117, M118	15.41 15.31	16.52 15.82	16.08 15.30	16.55 15.79	15.03 14.81	16.19 15.90	15.76 15.41	16.55 15.70	15.65 15.20	16.88 16.10
M137, M138	16.44 15.55	16.22 15.52	16.40 15.55	16.62 15.90	16.41 15.76	16.15 15.49	15.73 15.35	16.70 . . .	16.79 15.89	16.61 16.10
M149, M150					16.00 . . .	15.92 . . .		16.48 . . .		
M151, M152	16.63 . . .	15.58 . . .	16.82 15.81	15.94 . . .	15.01 . . .	16.59 . . .	16.43 . . .	15.90 15.36	16.58 . . .	16.82 . . .
M159, M160	16.80 15.77	16.19 15.53	15.71 15.08	. . . 15.76	(15.79) 15.26	16.64 15.96	16.79 15.96	16.49 15.83	16.99 16.21	. . . 16.13
M163, M164	15.56 15.11	16.19 15.70	16.08 15.30	16.79 . . .	16.24 15.52	15.63 16.38	16.99 16.20	16.70 . . .	15.68 15.28	16.18 . . .
M171, M172	16.28 15.67	16.18 15.78	16.21 15.45	(16.80) 16.13	15.49 15.03	15.71 15.81	16.73 15.88	16.22 15.60	17.01 16.31	16.29 . . .
M179, M180	15.76 15.12	16.63 15.65	16.35 15.62	(16.8) 15.89	16.06 15.47	15.50 15.04	16.24 . . .	16.60 15.90	15.87 16.26	16.5 16.00
M183, M184	16.14 15.44	15.31 14.89	16.81 15.70	16.00 15.28	16.32 15.61	15.93 15.28	15.70 15.29	17.10 . . .	16.33 15.70	16.7 16.0

TABLE III-Continued

Plates	HV 1987 3.130 ± .001	HV 1898 3.021 ± .001	HV 2027 2.984 ± .002	HV 1974 2.896 ± .001	HV 2000 2.880 ± .002	HV 1809 2.825 ± .002	HV 2049 2.800 ± .002	HV 1850 2.760 ± .002	HV 1981 2.724 ± .001	HV 1923 2.564 ± .001
M3, M4	16.52 16.01	16.35 16.10	16.96 16.28	17.17 16.37	17.22 16.28	17.07 . . .	16.74 . . .	17.00 . . .	17.00 16.27	16.45 15.9
M7, M8	16.69 16.20	16.47 16.03	17.62 16.40	(15.78) 15.44	16.67 16.05	17.30 16.80	17.14 16.51	17.12 15.64	17.75 (16.70)	17.07 16.9
M11, M12	. . . 15.69	16.38 . . .	16.96 16.23	17.09 16.25	17.16 16.29	17.15 . . .	16.73 15.82	16.92 . . .	17.20 (16.50)	16.87 16.17
M21, M22	. . . 16.37	. . . 15.51	16.24 15.40	. . . 15.94	16.31 15.90	. . . 15.69	. . . 15.74	. . . 15.90	. . . 15.84	16.0 15.58
. . . M26	. . . 15.70	. . . 15.93	. . . 16.13	. . . 16.48	. . . 16.41	. . . 16.37	. . . 16.02		. . . 16.45	. . . 16.41
M31, M32	16.99 16.20	15.82 15.51	16.10 15.58	17.35 16.18	17.09 16.24	17.17 16.40	17.15 16.30	15.60 15.43	. . . 16.79	15.95 15.73
M37, M38		16.57 16.11	17.70 16.25	16.57 15.85	16.40 15.95	16.54 16.10	16.72 15.82	17.12 15.90	17.50 15.45	16.71 (16.51)
M41, M42	16.85 16.20	16.03 15.62	16.40 15.67	17.30 16.48	17.20 16.31	(17.50) 16.23	16.24 . . .	16.45 15.69	17.65 (16.98)	16.87 16.20
M49 . . .	16.96 . . .	16.36 . . .				16.00 . . .	16.11 . . .			
M55, M56		16.50 . . .			(16.10) . . .			15.82 15.25		16.69 . . .
M61, M62		15.95 . . .	16.44 15.72	17.30 . . .	17.20 16.65			16.36 16.02	16.44 . . .	16.99 . . .
M69, M70	16.38 15.83	15.87 . . .	16.39 15.59	17.3 . . .	16.99 (16.25)	15.85 . . .	16.30 . . .	16.76 . . .		17.11 . . .
M73, M74	17.22 . . .	16.48 . . .	17.74 . . .	16.40 . . .	17.09 . . .	17.07 . . .	17.07 . . .	17.02 . . .	17.60 . . .	17.49 . . .
M81, M82	16.02 15.69	16.20 . . .	17.11 16.25	17.13 (16.00)	17.13 16.40	(16.64) 15.60	16.46 15.59	15.78 15.35		17.42 . . .
M89, M90		(16.45) . . .	16.60 . . .	16.34 . . .	16.34 . . .			17.00 . . .		
M93, M94	16.90 16.10		16.59 (15.95)	16.65 . . .	(16.68) 15.87		. . . 16.55	17.10 . . .	17.70 . . .	17.0 (16.40)
M101, M102		16.49 . . .	17.0 . . .	16.30 . . .	(15.96) . . .			(16.53) . . .		(15.95) . . .
M107, M108	16.50 15.98	16.46 16.10	17.28 16.93	15.80 15.67	15.98 15.65	17.09 16.28	17.29 16.50	17.22 16.20	17.57 16.99	17.02 16.44
M113, M114	16.71 16.09	16.57 16.08	16.43 16.30	16.80 16.11	17.04 16.27	17.21 16.53	16.25 15.92	16.09 15.37	16.33 16.15	16.83 16.04
M117, M118	16.80 15.90	16.31 15.75	17.05 16.93	17.20 16.55	17.24 16.43	16.55 15.91	16.99 16.37	16.65 16.09	17.40 16.50	16.81 16.20
M137, M138	17.09 16.29	16.10 15.68	17.48 16.40	17.23 16.50	15.92 15.63	16.09 15.64	17.01 16.35	15.71 14.40	17.20 16.29	16.38 15.95
M149, M150				(16.15) . . .	. . . 15.63				16.50 . . .	
M151, M152	17.25 . . .	16.30 15.99	16.28 . . .	17.15 . . .	17.19 . . .	16.12 15.55	(17.28) . . .	16.42 . . .	17.78 . . .	16.61 . . .
M159, M160	15.91 15.45	16.50 16.15	17.17 16.20	(17.30) 16.23	16.08 15.73	. . . 16.13	16.27 15.73	16.26 . . .	16.38 16.01	17.26 16.90
M163, M164	16.64 15.26	15.87 . . .	17.50 . . .	16.50 15.59	. . . 16.30		16.90 . . .	16.70 16.08	. . . 16.35	15.89 15.80
M171, M172	17.10 16.20	16.58 . . .	17.26 16.39	16.10 15.65	. . . 16.05	(17.11) 16.40	16.98 . . .	16.30 15.70	. . . (17.20)	17.67 16.6
M179, M180	15.85 15.50	16.08 15.70	17.72 16.50	16.71 16.09	. . . 16.34	16.15 15.80		16.85 16.08	16.31 15.95	16.24 15.83
M183, M184	16.70 15.83	16.39 15.76	16.34 15.79	17.06 16.26	15.85 15.62	(17.22) . . .	16.28 15.78	16.75 16.16	17.40 16.42	17.15 16.30

TABLE III-Continued

Plates	HV 2046 2.554 ± .001		HV 850 2.5350 ± .0007		HV 2021 2.4875 ± .0010		HV 1192 2.3543 ± .0007		HV 2002 2.345 ± .001		HV 844 2.219 ± .001		HV 2014 2.2028 ± .0008		HV 848 2.178 ± .001		HV 1199 2.0952 ± .0009		HV 2035 1.9800 ± .0005	
M3, M4	16.08	16.0	17.39	16.44	16.66	16.24	17.55	. . .	16.07	15.88	17.22	. . .	17.17	16.36	16.59	. . .	16.63	16.11	16.42	16.11
M7, M8	. . .	16.46	16.93	15.85	17.32	16.75	16.90	16.49	16.78	16.18	. . .	15.83	16.48	16.01	17.45	16.59	16.30	15.9	17.54	16.5
M11, M12	16.50	15.82	16.00	15.61	17.14	16.43	16.25	. . .	16.39	15.93	17.56	. . .	15.88	16.37	17.28	. . .	16.11	15.80	17.46	16.83
M21, M22	. . .	15.64	. . .	15.39	16.69	15.92	. . .	16.67	. . .	15.97	. . .	. . .	16.00	. . .	16.31	. . .	. . .	15.74	(17.13)	16.8
. . . M26	. . .	16.09	. . .	16.17	. . .	16.63	. . .	16.34	. . .	16.38	. . .	16.25	. . .	16.05	. . .	16.72	. . .	15.98	. . .	16.75
M31, M32	16.31	15.8	17.33	16.53	17.12	16.50	(17.28)	16.80	17.47	16.55	17.47	. . .	16.25	15.82	17.59	16.66	16.62	15.99	17.52	16.75
M37, M38	17.27	15.69	17.39	. . .	16.82	16.02	(17.14)	16.15	. . .	. . .	(17.38)	. . .	16.02	15.21	17.53	. . .	16.65	16.00	17.70	16.98
M41, M42	16.46	16.04	16.34	16.08	17.20	17.00	. . .	16.75	16.13	15.70	17.00	16.20	16.20	16.40	16.20	16.01	16.23	15.80	16.68	16.26
M49 . . .	17.26	. . .	. . .	. . .	. . .	. . .	17.18	. . .	(17.18)	15.90	. . .	. . .	16.80	. . .	. . .	. . .	16.45	. . .	. . .	. . .
M55, M56	16.58	. . .	16.19	. . .	. . .	. . .	15.74	. . .	. . .	. . .	. . .	. . .	. . .	. . .	16.97	. . .	. . .	. . .	. . .	. . .
M61, M62	17.09	. . .	17.20	16.60	16.24	. . .	. . .	. . .	17.21	. . .	15.91	. . .	16.92	. . .	17.03	. . .	16.70	. . .	16.53	. . .
M69, M70	. . .	. . .	. . .	. . .	16.94	. . .	15.78	15.60	16.60	. . .	15.78	. . .	16.71	. . .	16.31	(16.13)	16.49	. . .	16.54	16.25
M73, M74	17.00	. . .	17.32	. . .	16.96	. . .	16.46	. . .	16.67	. . .	16.82	. . .	17.3	17.11	. . .	. . .	16.81	. . .	17.51	. . .
M81, M82	(17.30)	. . .	16.35	. . .	17.15	16.25	. . .	. . .	16.41	. . .	(17.50)	. . .	15.99	15.59	16.82	(17.05)	16.78	16.22	16.80	(16.95)
M89, M90	16.45	. . .	16.86	. . .	16.81	. . .	. . .	. . .	16.02	. . .	. . .	. . .	16.50	. . .	(17.4)	. . .	(16.70)	. . .	. . .	. . .
M93, M94	17.10	. . .	17.11	. . .	17.31	16.60	15.91	. . .	. . .	(16.20)	(16.99)	. . .	16.41	(16.40)	16.39	15.94	16.40	. . .	17.49	17.10
M101, M102	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
M107, M108	17.05	16.25	17.25	16.32	17.57	16.59	17.14	16.28	(17.31)	. . .	17.33	16.38	16.25	16.00	17.19	16.33	16.41	15.90	17.57	17.20
M110, M114	16.45	15.95	16.93	15.35	16.70	16.09	16.70	16.80	16.29	16.00	16.70	16.30	16.00	16.55	17.04	(16.35)	16.51	15.99	16.99	16.80
M117, M118	16.99	(16.50)	17.00	16.25	17.52	(16.78)	16.70	16.09	. . .	16.40	16.92	16.23	(17.60)	. . .	17.50	16.72	16.65	15.86	17.35	17.05
M137, M138	16.79	16.35	17.37	16.50	17.12	16.50	16.11	15.69	(17.15)	16.44	. . .	16.39	16.10	15.78	17.40	16.22	16.39	16.05	17.59	16.85
M149, M150	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	16.10	. . .	. . .	. . .	. . .	. . .	. . .	. . .
M151, M152	(17.30)	. . .	15.74	. . .	17.69	. . .	16.82	16.00	. . .	. . .	17.20	16.30	16.81	. . .	17.70	. . .	16.78	15.60	16.45	. . .
M159, M160	. . .	. . .	17.03	16.34	16.70	16.20	. . .	16.58	16.27	15.71	. . .	16.62	. . .	16.50	17.00	16.19	16.60	15.74	17.85	16.85
M163, M164	16.65	(16.35)	>17.5	16.70	. . .	16.83	15.90	. . .	(17.02)	(16.40)	16.49	. . .	15.87	15.70	(17.45)	. . .	16.67	. . .	16.47	16.40
M171, M172	16.01	. . .	17.25	16.67	17.42	16.90	(17.09)	16.20	16.10	. . .	16.65	16.05	16.01	. . .	(17.35)	16.44	16.51	(15.8)	16.55	16.05
M179, M180	17.1	16.27	15.91	15.40	. . .	16.80	16.60	16.35	16.85	. . .	. . .	. . .	(17.18)	. . .	16.59	15.88	16.5	. . .	. . .	16.80
M183, M184	16.90	16.29	16.97	16.11	16.95	16.35	(17.20)	16.20	17.16	16.58	16.40	15.75	15.75	15.64	(17.66)	(16.85)	16.41	15.69	16.59	16.80

TABLE III -Continued

Plates	Arp a 1.6586 ± .0004	HV 2010 1.6303 ± .0004	HV 11198 1.6180 ± .0004	HV 2022 1.3090 ± .0005	HV 11197 1.0736 ± .0002	Arp b (2.078) ?	HV 1978 2.7	HV 1868 2.8	HV 1836 2.9	Arp g 4-5
M3, M4	16.80 16.5	18.20 17.02	17.15 16.75	17.0 16.8	16.52 16.07	17.35	16.51 16.00	16.87	16.21	15.3
M7, M8	18.3	17.77 16.85	17.01 (16.5)	18.4 >17.0	16.39 16.50	16.65 16.41	16.17 15.94	16.89 (16.73)	17.05 (16.40)	15.3 15.39
M11, M12	18.20 17.3	17.71	17.30 (16.8)	18.0 17.4	17.2 (16.69)	17.33	16.07 15.91	16.63 (17.15)	16.07 15.76	15.4 15.19
M21, M22	17.07 16.50	(16.2)	(16.80)	18.4 17.6	17.0	(17.7)	15.86	16.07	16.56	15.2 15.17
M25	17.3	16.81	(16.4)	17.4	16.7	16.97	15.82	16.84	15.79	15.85
M31, M32	(18.25) (17.31)	(18.10) (17.33)	17.35 16.65	18.2 17.5	17.10 16.85	17.10 16.64	16.21 15.71	16.77	16.57 16.01	15.4 15.22
M37, M38	17.00	18.20 16.59	16.53 16.20	17.0 16.6	16.95	17.38 (16.51)	16.20 15.38	16.93 (16.51)	16.71 (16.24)	15.3 15.49
M41, M42		17.99 17.10	17.25 (16.90)		16.80	16.91 (16.55)	16.35 15.94	16.82 (16.05)	16.77 15.90	15.12 15.13
M49	17.0			18.2	16.80			(16.35)	16.92	(15.15)
M55, M56								17.04	16.77	15.11 (15.20)
M61, M62		17.24 (16.50)	17.30		17.04		17.12	16.45	16.51 15.55	(15.71) 15.40
M69, M70	(17.03) (16.80)	17.34 16.86		(16.95)		17.08		16.88	16.61	15.68
M73, M74	17.0 (16.75)	17.53	17.25	(17.5) >17.0	16.59	>17.3	16.75	16.82	16.92	15.41
M81, M82	16.75 16.5	17.61		>18.3	(17.29)	17.15 >17.0	16.30	16.88	16.72 16.30	15.30 15.12
M89, M90	>17.3		16.9	16.7		16.95	16.36	16.31	>17.3	15.15 (15.07)
M93, M94	(16.8) (16.4)	(17.74) (17.00)	17.0 (16.7)	17.4		17.30 >17.0	16.20 15.41	16.80	(16.72)	15.10 14.89
M101, M102									>16.3 (15.05)	(15.4) . . .
M107, M108	18.2 17.01	17.45 16.96	17.25 16.55	>18.4 17.35	17.25 17.00	(17.5) 16.80	16.34 15.59	16.47 (17.05)	16.78 16.27	. . . 15.27
M113, M114	18.25	16.30 16.29	17.32 16.72	17.3 17.1	17.24 17.10	17.57 16.84	16.90 15.76	16.79 16.57	16.67 15.90	15.14 15.01
M117, M118	16.9 16.49	18.00 17.17	16.5 16.20	17.2 17.2	17.30 16.50	16.79 16.40	16.65 15.80	16.72 (17.23)	16.58 15.89	15.15 15.09
M137, M138	17.07 16.50	17.38 16.77	16.67 16.24	17.5	16.40 16.00	17.04 16.25	16.53 15.98	16.99 16.69	(17.11) 16.05	15.28 15.27
M140, M150									16.00	15.25
M151, M152	18.30	17.55	16.99	16.75	(17.15) 16.10	16.60 16.15	16.61	16.99	16.73	15.7 16.50
M159, M160	(18.3)	17.92 16.79	16.95 (16.74)	18.5 17.5	(17.25) 16.72	17.85 (17.38)	17.19 15.64	16.89 16.80	16.43 16.13	15.55 15.55
M163, M164		17.20		>18.8		17.16	16.90	16.83	16.62	15.19 15.29
M171, M172	18.15 (17.35)	(18.05) 17.20	16.8	18.25 (17.4)	16.80	16.9 16.60	17.05 15.75	16.69 16.60	17.22 16.14	15.38 15.32
M179, M180	(18.3)		17.35 (16.45)	>18.2 >17.0	16.56	17.3 16.30	15.98 15.65	17.05 16.08	16.56 15.35	15.28 15.24
M183, M184	16.75 16.4	16.25 16.02	16.7 16.12	17.5 16.9	16.59	17.4 16.59	16.43 15.71	16.91 16.62	16.46 15.91	15.3 15.01

panion plate was taken in the ultraviolet, and M 49 where the companion plate was not measured. In general the longer period cepheids, and hence the brighter, were measurable on every plate. Toward the end of Table III the fainter cepheids are not measurable on many plates, particularly in the middle of the observing span when the SMC was lowest in the sky. At that time the seeing was poorer on the average. Also, exposures at that time had to be limited to less than one hour, whereas they averaged more than an hour in the beginning and end of the observing periods. Some plates were offset toward other SMC regions, and only variables in certain parts of the cepheid field were measurable.

All the cepheids in Table III and all the standard stars in Table I were measured on each plate. Generally the outlying standards indicated calibration curves which were from 0 to 0.3 mag. shifted from the primary, central calibration curve. As mentioned in Sec. 4, there were both systematic and random shifts. These were only the outermost variables, however, and they were reduced by the calibration curves of the nearest standards, which had been placed there for that purpose.

When all the magnitudes had been measured and reduced, it was found that the cepheids in the more central regions, those say from 15' to 25' from the center, showed systematic deviations on some plates. On roughly half the plates all the cepheids in a certain region would be too bright or too faint (as judged relative to their mean light curves) by an amount up to 0.2 mag. This inner 25' radius circle which contained the majority of the cepheids was then broken into three subregions plus a small region around the central primary sequence. On plates which showed the effect, the mean correction to each subregion was determined and the magnitudes reduced a second time. This iterative procedure has produced individual magnitude measures

which are accurate to better than ± 0.1 mag. (as estimated from the scatter about the best light curves).

8. The Periods

In the case of each variable listed in Table I, phases were initially computed with the Harvard period. These periods are as published by Shapley *et al.* (1940), McKibben *et al.* (1942), and Nail (1949). A summary of the available Harvard periods is given in Table IV, where some more accurate periods from Shapley *et al.* (1947) are also included. A chronological bibliography of the publication of Harvard periods in the SMC is given by Shapley and Nail (1955).

The phases computed with the original Harvard periods were used to construct light curves for each variable. These light curves represented the observations during the eight-month span of the present observations. The original period was only changed if the observations in the latter part of this time span were systematically shifted from the observations in the early part of the span. The accuracy with which the beginning and ending observations could be matched was estimated. This determined the accuracy with which the period was known from the present set of observations alone. The final periods derived for the present group of cepheids during the interval November, 1955, to June, 1956, are given at the head of each column in Table III and summarized in the next to last column of Table IV.

The accuracy of some light curves was greater than others, and in some cases the late and early observations fell in critical portions of the light curve (say the ascending branch) where small systematic phase differences could be more accurately detected. In such cases the accuracy of the derived period was greater than in cases where the light curve was less well defined. But in general the accuracy with which the

TABLE IV. Periods of cepheids in SMC.

Variable HV	H.C. 280	H.B. 900	H.C. 439	H.C. 444	H.A. 109	H.B. 919	Arp (1955-1956)
1877		49.6	49.5				49.5 ± 0.3
837	42.6		42.6				42.6 ± 0.4
11182				39.6			39.6 ± 0.2
2064		33.7	33.7				33.7 ± 0.3
840		33.1	33.1				33.1 ± 0.2
1967		29.1	29.1				29.1 ± 0.1
847	27.2		27.2				27.2 ± 0.1
1884		18.1	18.11				18.11 ± 0.04
1925		17.2	17.18				17.18 ± 0.07
1954		16.7	16.71				16.71 ± 0.02
1787	16.22		16.22				16.22 ± 0.04
843				14.66			14.70 ± 0.02
1695						14.512	14.50 ± 0.05
1933		13.8	13.80				13.76 ± 0.04
1873		12.9	12.91				12.91 ± 0.02
1744		12.6	12.59				12.59 ± 0.03
856		12.2	12.19		12.155		12.13 ± 0.03
2017		11.4	11.40		11.4072		11.43 ± 0.02
2063				11.25	11.166230		11.18 ± 0.02
2060		10.2	10.21		10.18447		10.21 ± 0.01
1768	9.86		9.86		9.808210		9.798 ± 0.01
1950				7.98			7.980 ± 0.01
1905				7.41			7.413 ± 0.010
1855				6.81	6.83990		6.830 ± 0.006
1945				6.47			6.465 ± 0.005
11193				6.44			6.440 ± 0.008
1858	6.15		6.15				6.120 ± 0.007
1892				5.66			5.660 ± 0.005
1929				5.60			5.579 ± 0.007
1903						5.1028	5.091 ± 0.004
1827	3.60		3.60				4.924 ± 0.002
1934				4.89			4.885 ± 0.003
1785	4.90		4.90				4.725 ± 0.003
1966				4.285			4.270 ± 0.002
1825	4.23		4.23				4.265 ± 0.003
1994				1.309			4.217 ± 0.003
1793				4.19			4.180 ± 0.004
1891				3.45			3.451 ± 0.002
11206				3.40	3.399471		3.400 ± 0.002
1987				3.13			3.130 ± 0.001
1898				3.03			3.021 ± 0.001
2027				2.985			2.984 ± 0.002
1974				2.88			2.896 ± 0.001
2000				2.88			2.880 ± 0.002
1809				2.825			2.825 ± 0.002
2049				2.80			2.800 ± 0.002
1850	2.767		2.767				2.760 ± 0.002
1981				2.722			2.724 ± 0.001
1923				2.565			2.564 ± 0.001
2046				2.553			2.554 ± 0.001
850				2.535			2.5350 ± 0.007
2021				2.489			2.4875 ± 0.0010
11192				2.355			2.3543 ± 0.0007
2002	0.702			2.345*			2.345 ± 0.001
844				2.218			2.219 ± 0.001
2014				2.203			2.2028 ± 0.0008
848	2.178		2.178				2.178 ± 0.001
11199				2.104			2.0952 ± 0.0009
2035						1.97945	1.9800 ± 0.0009
Arp a							1.6568 ± 0.0004
2019				1.629			1.6303 ± 0.0004
11198				(1.618)			1.6180 ± 0.0004
2022						1.30887	1.3090 ± 0.0005
11197				1.074			1.0736 ± 0.0002

* H.R. 270.

period could be determined depended primarily on the period itself. If ϕ is the phase, T the interval of observation, and P the period,

$$\phi = T/P \quad (5)$$

$$\Delta P = (-P^2/T)\Delta\phi. \quad (6)$$

The phase of the final observations could be set with an uncertainty, $\Delta\phi$, which, in this study, ranged from ± 0.02 to ± 0.10 . T , in this study, represented the span of observation time shown in Table III—usually about 30 days.

Some of the cepheids seem to have undergone significant period changes, but they will be discussed at the end of Sec. 26.

9. The Light Curves

Light curves of 64 cepheids are shown in Figs. 3 through 7. Individual points, as taken from Table I, are plotted for each variable. A mean line is drawn through these points. Of course, this mean line is uncertain in some parts of some curves, but it is drawn in all cases as a record of the assumptions underlying the derived data which follows. Notes on specific variables are given at the bottom of Table V.

The B light curve is given for each variable with the magnitude scale at the left. Below each B curve is the V curve with the magnitude scale to the right. The light curves have been drawn through the points as best possible, giving consideration to the reliability of individual measures, the general behavior of the light curve, and the random fluctuation of the points. Sometimes a feature will appear in the B curve which does not appear in the V curve, and vice versa. Some of these discrepancies may be simply the result of random errors or they may be partially due to occurrence of different features with different phases or different amplitudes in the two colors.

A few light curves published by Harvard (Shapley 1940; Shapley, Carlston, and Nail 1947) are of the same stars as measured here. (There are three in the first reference and seven in the second.) It is very interesting to note that the *general* character of the light curves, such as asymmetry and relative amplitude, agree very well, but even the most well-founded bumps, humps, and irregularities in the light curves are not repeated at all between the two investigations.

The conclusion from the available data must be that any secondary features of the light curves which are real must have changed between the epochs of the two investigations.

10. The Light-Curve Parameters

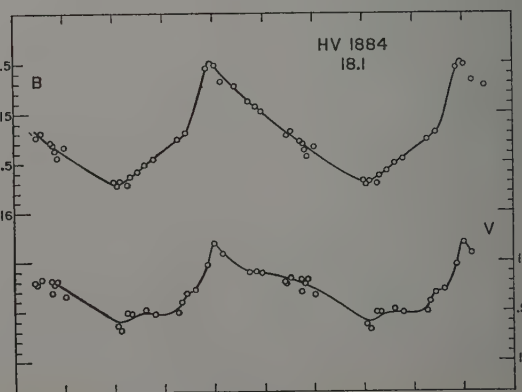
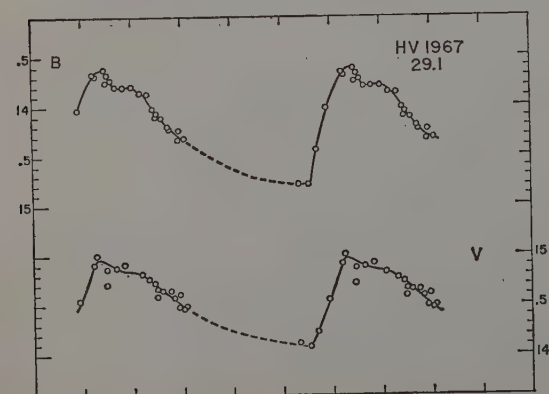
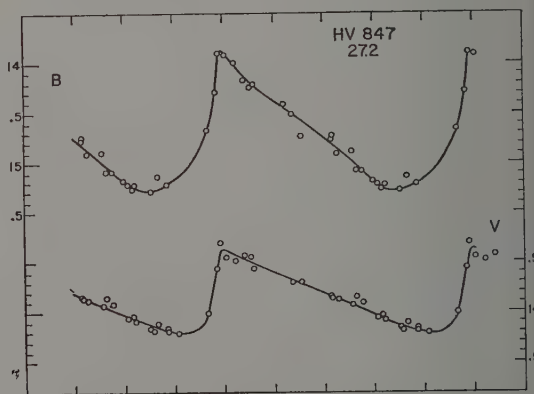
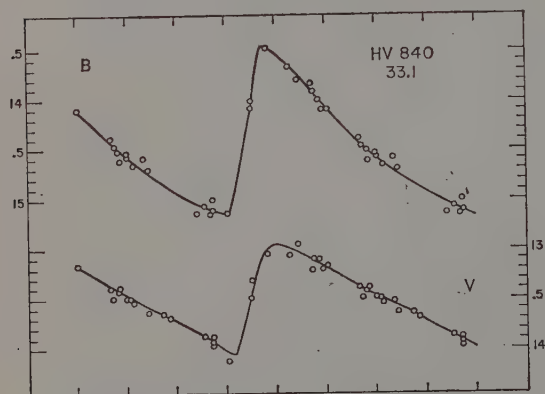
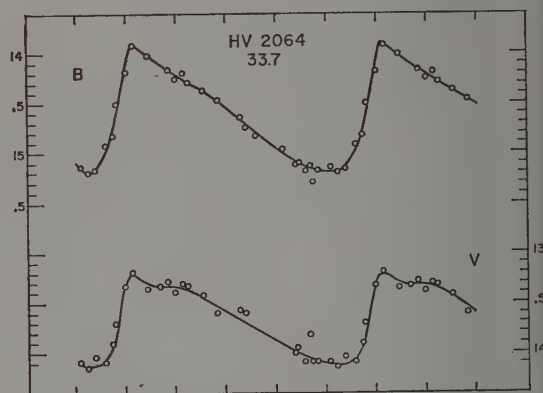
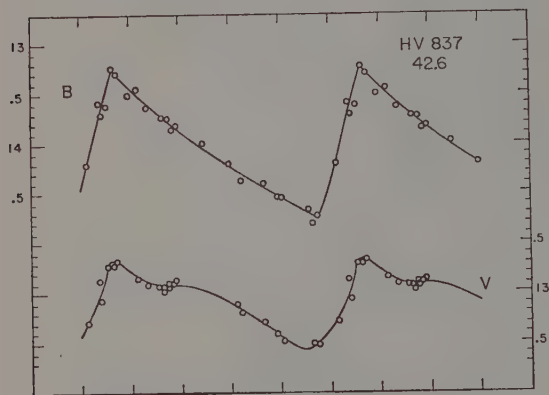
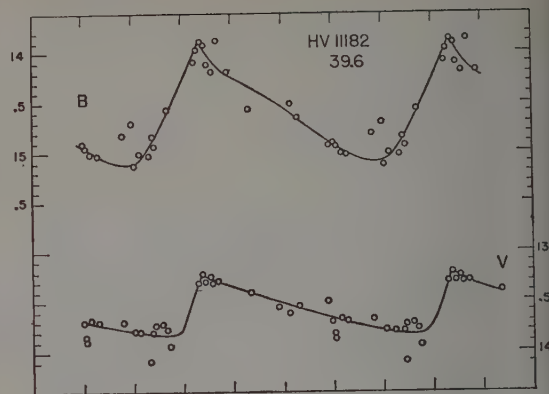
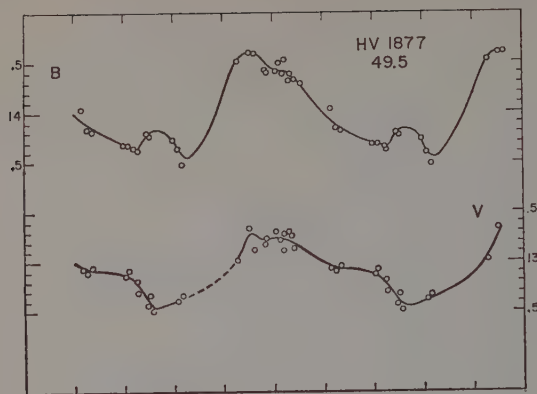
Table V lists all the primary data derived from the light curves. The maximum and minimum values were read directly from the light curves pictured in Figs. 3 through 7.

The mean magnitudes listed as B_{mean} and V_{mean} in Table V were obtained by planimetry of the *magnitude* curves of the variables. (Actually only a few light curves were planimetryed since it was found that a much quicker method, that of laying a thin straight edge across the light curve and adjusting until the upper area balanced the lower, gave mean magnitudes accurate to a few hundredths of a magnitude.)

The parameter of mean magnitude has an advantage over say, median magnitude, because it is not subject to irregularities in the light curves at maximum and minimum. It differs slightly however, from the physically significant quantity—magnitude at mean light intensity. This latter quantity was obtained for five variables listed in Table VI. In all light curves the stretching of maximum relative to minimum in the transformation to intensity yields a brighter mean intensity-brightness. This correction is primarily a function of amplitude. This is shown in Fig. 8 where the differences in Table VI have been plotted as a function of amplitude. The difference between magnitude at mean light and mean magnitude is secondarily a function of the light-curve shape (whether the variable spends a short or long time near maximum). A sample of different shapes was chosen to investigate this effect. Figure 8 shows by the scatter of the points within the band that the effect of different light-curve shape is only ± 0.02 mag. at most for the intensity correction.

In the present paper all the variables were not transformed to an intensity scale and planimetryed because of the large amount of work involved. Rather, the B_{mean} and V_{mean} of Table V were converted to $\langle B \rangle$ and $\langle V \rangle$ (these symbols designate intensity means) by means of the correction, as a function of amplitude, as given by Fig. 8. The $\langle B \rangle$, $\langle V \rangle$ intensity-mean magnitudes so derived incur small errors due to different light-curve shapes. These errors are, however, at the most ± 0.02 mag., and therefore all the mean quantities referred to in the following tables and diagrams refer quite accurately to mean light intensity.

The quantity of asymmetry listed in the next to the last column simply measures the fraction of the light curve which is occupied by the descending branch of the light curve. This is a rather standard measure of the "skewness" of the light curve and is important because it is an index related to how anharmonic the pulsation is. In many light curves, however, the minimum is extremely flat and therefore the duration of the descending branch, and therefore the quantity of asymmetry, is uncertain. In an attempt to get at this same "skewness" quantity in another and possibly more accurate manner, the rate-of-rise was measured. This quantity is given in the last column of Table V. The rate-of-rise was obtained by measuring the average slope of the rising branch of the light curve. It is expressed in phase units required to rise 1 mag. This quantity, too has its disadvantages because some light curves show sudden change in slope part way up to



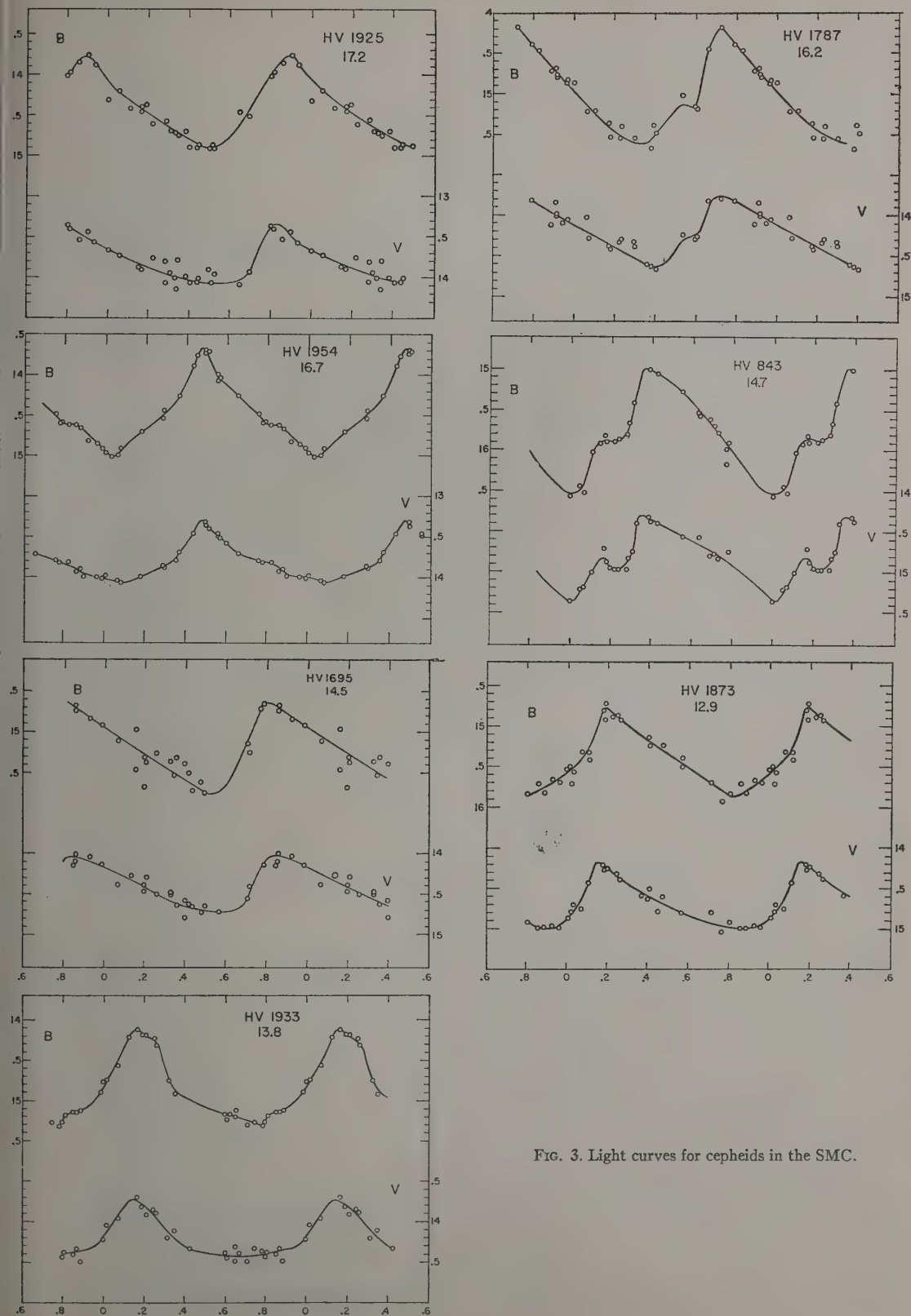
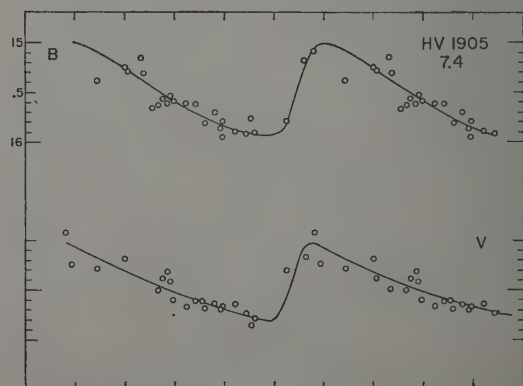
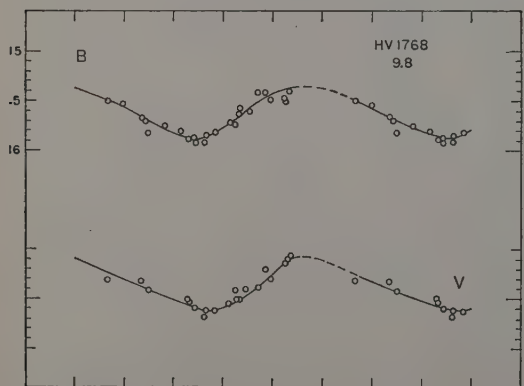
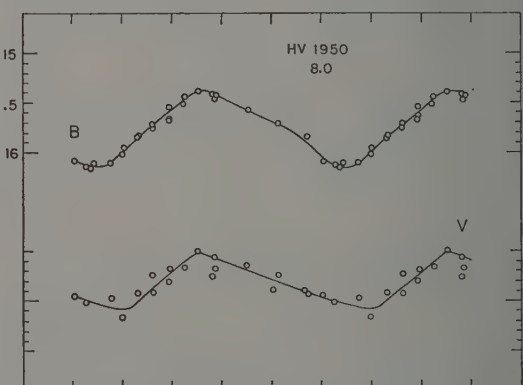
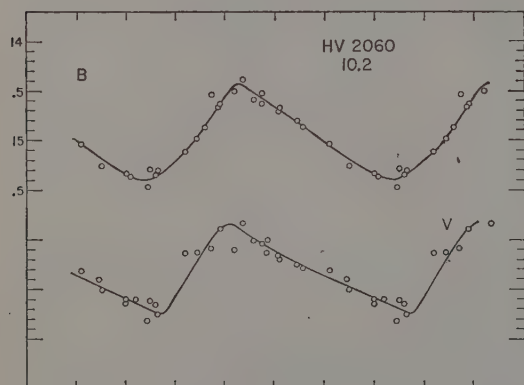
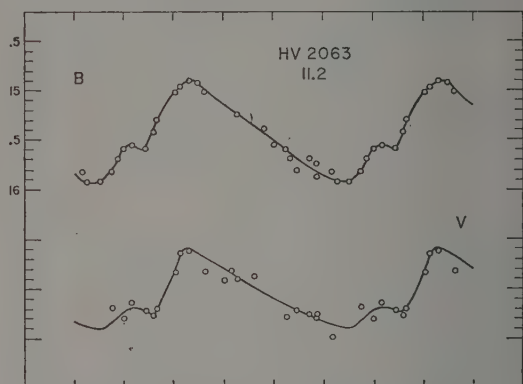
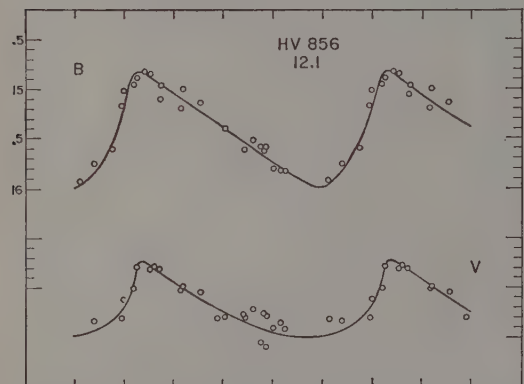
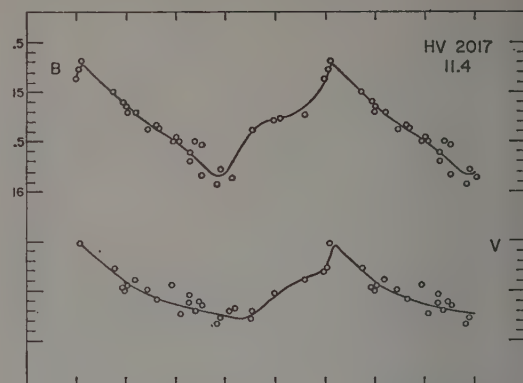
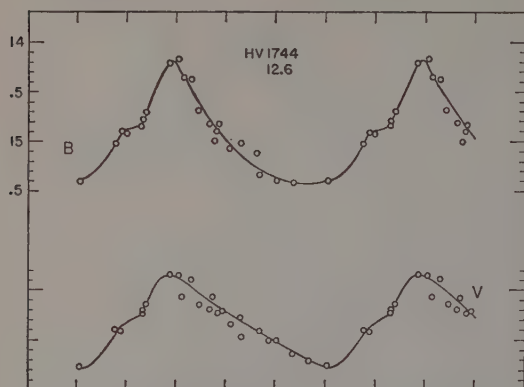


FIG. 3. Light curves for cepheids in the SMC.



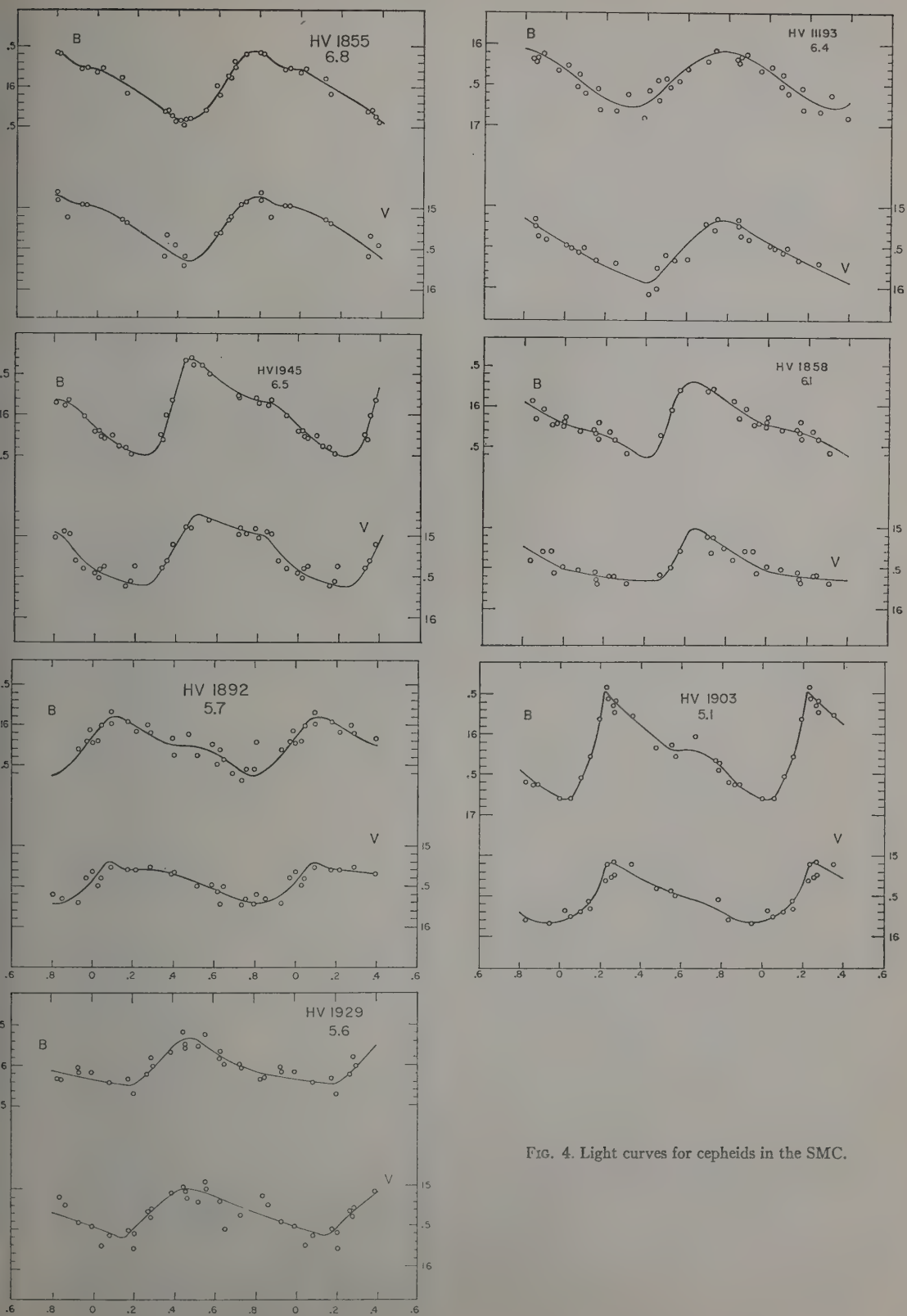
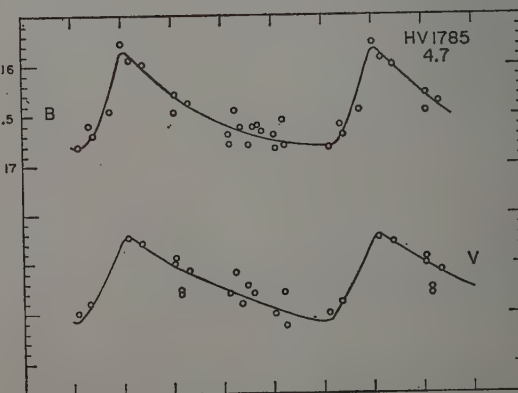
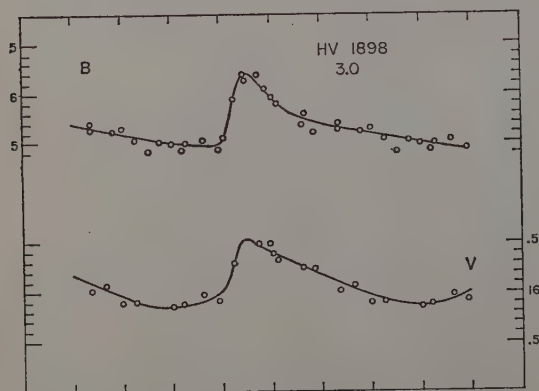
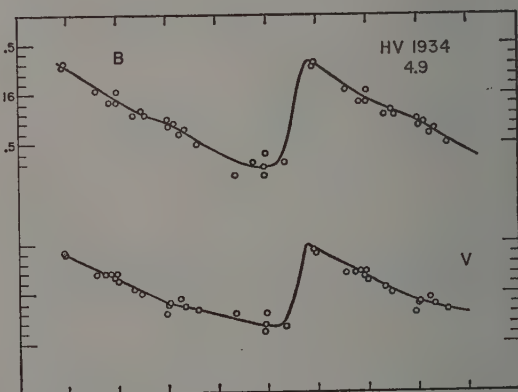
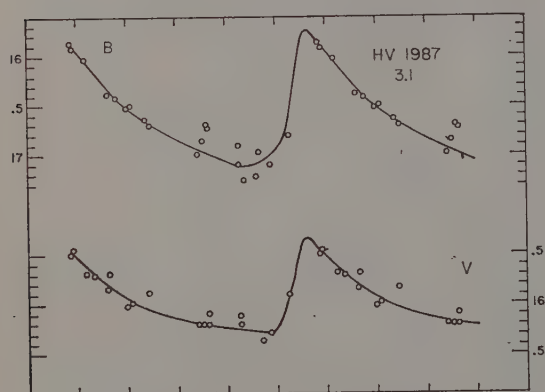
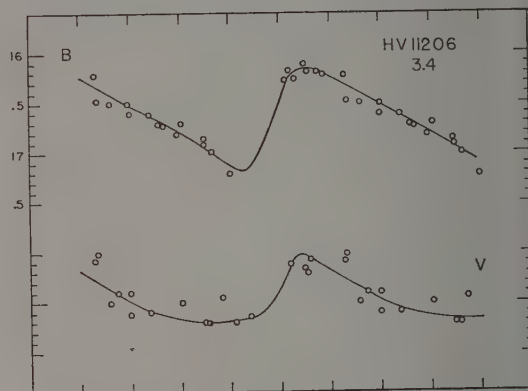
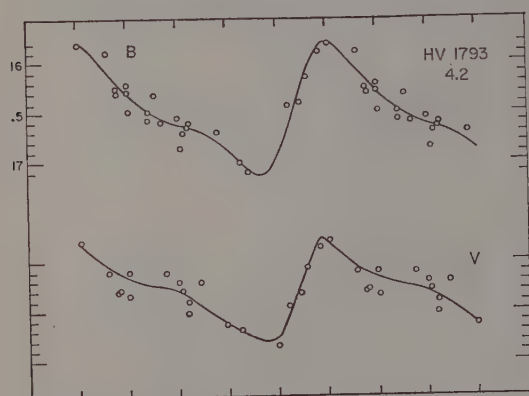
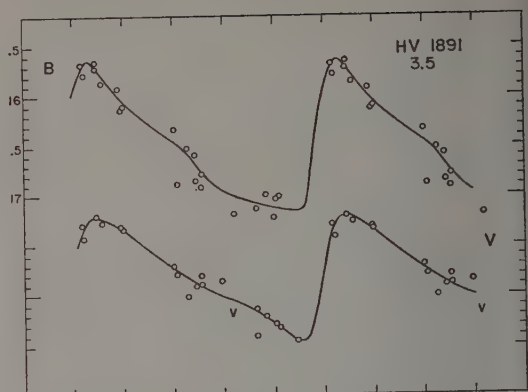
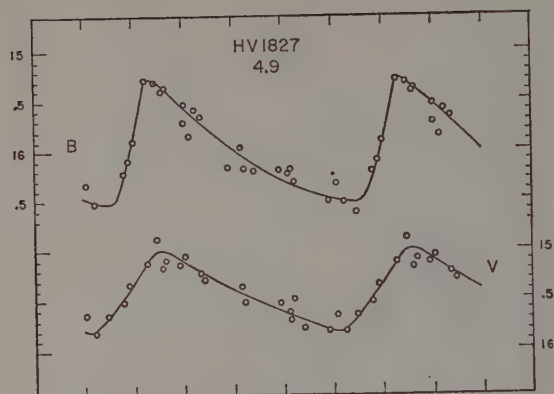


FIG. 4. Light curves for cepheids in the SMC.



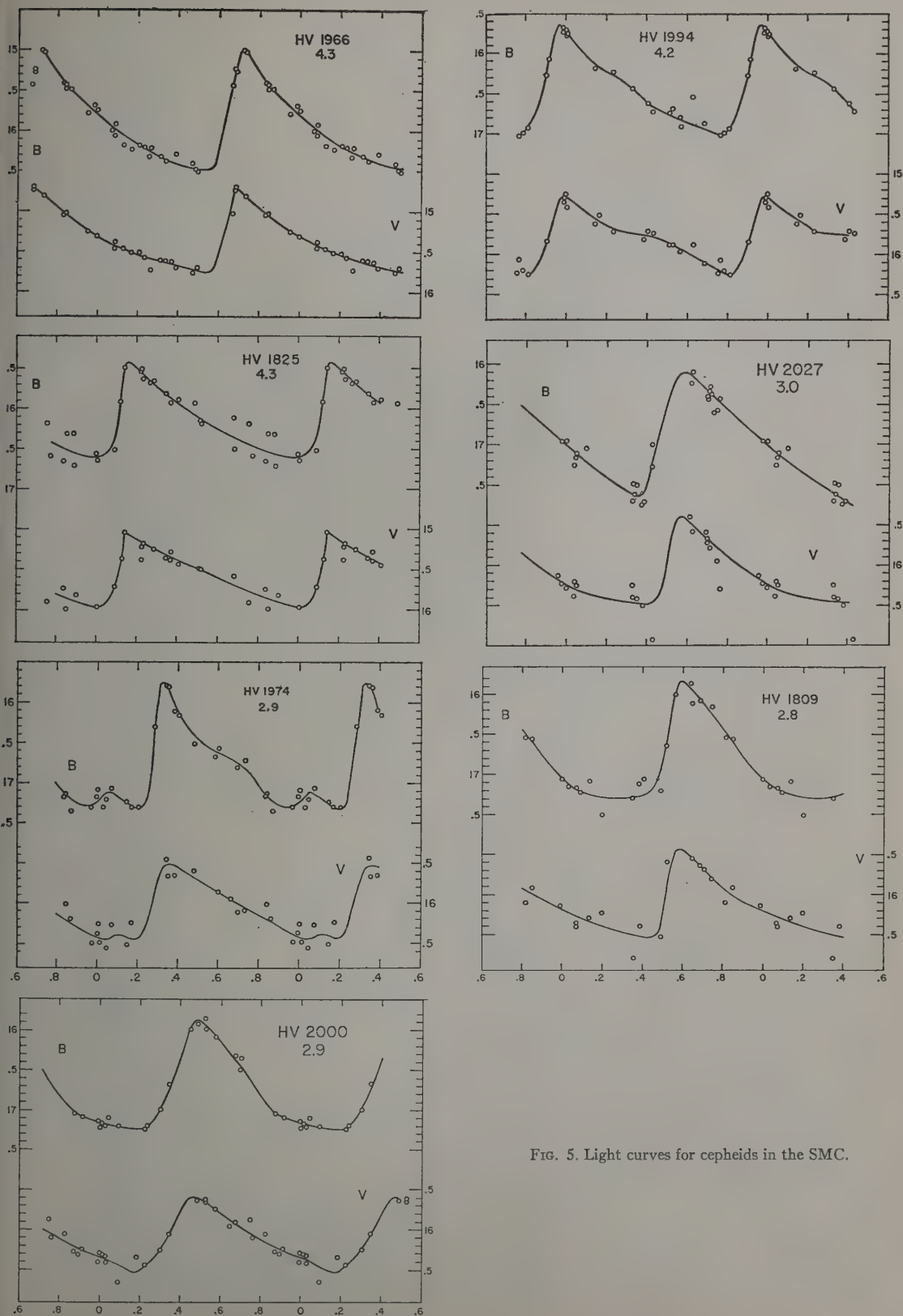
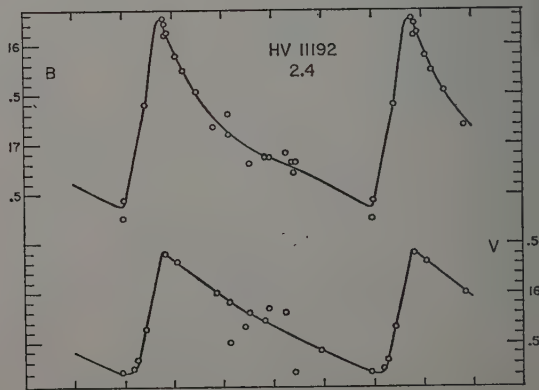
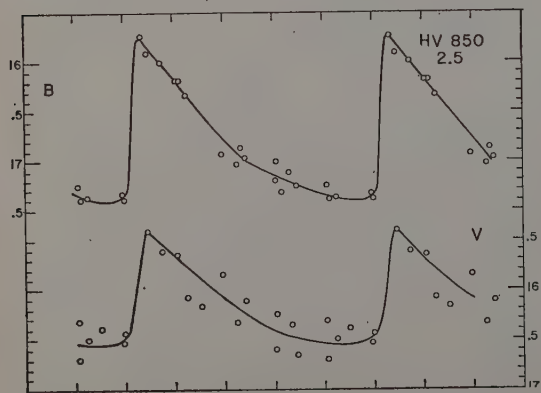
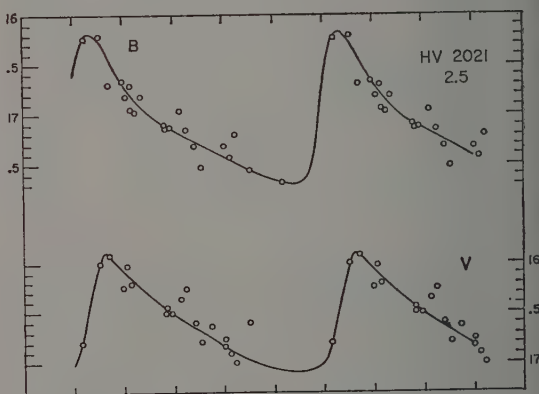
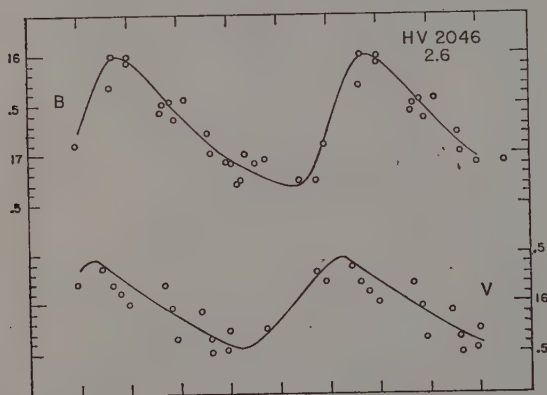
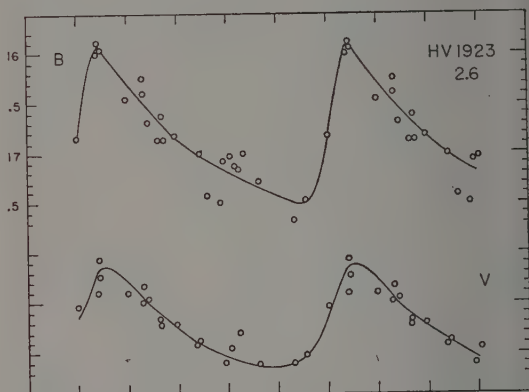
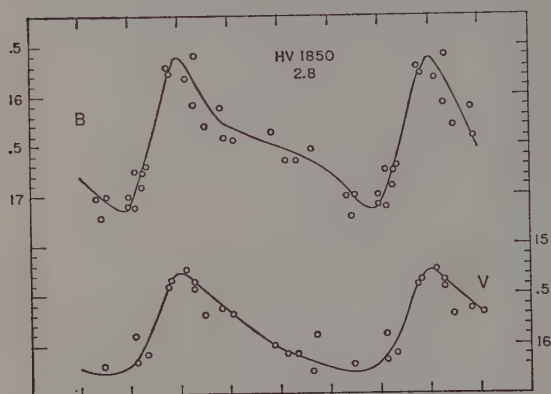
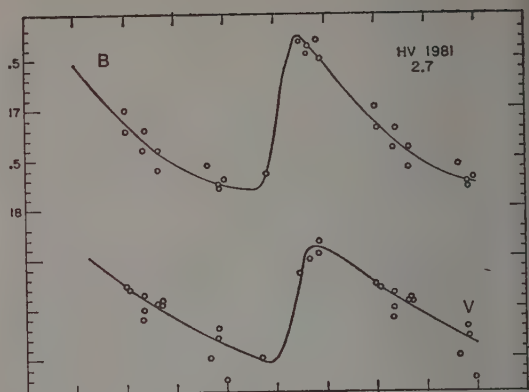
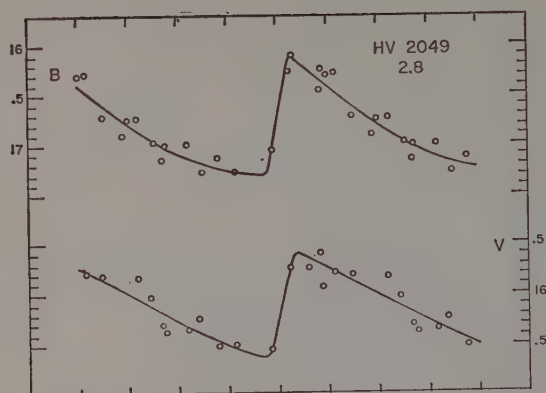


FIG. 5. Light curves for cepheids in the SMC.



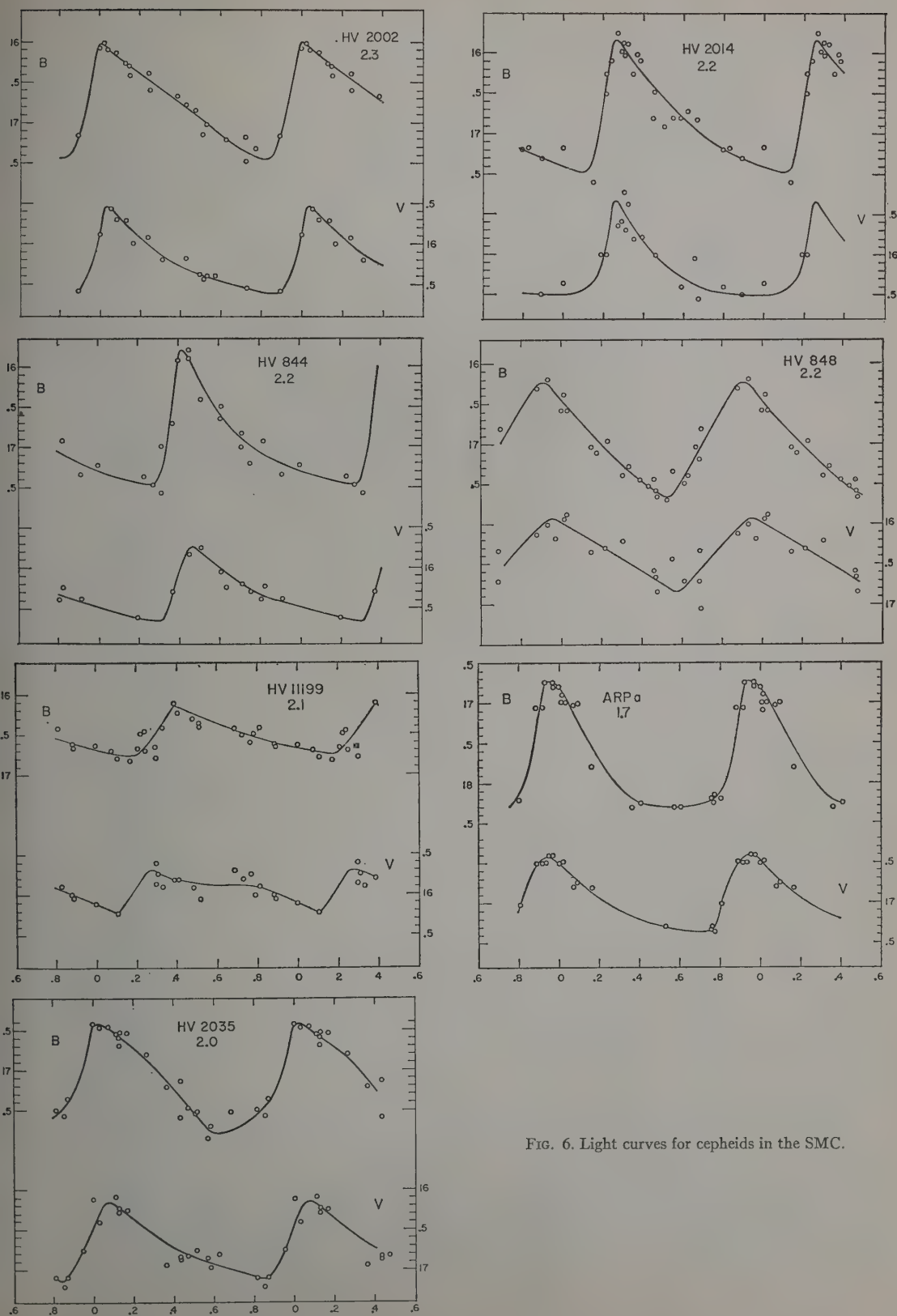


FIG. 6. Light curves for cepheids in the SMC.

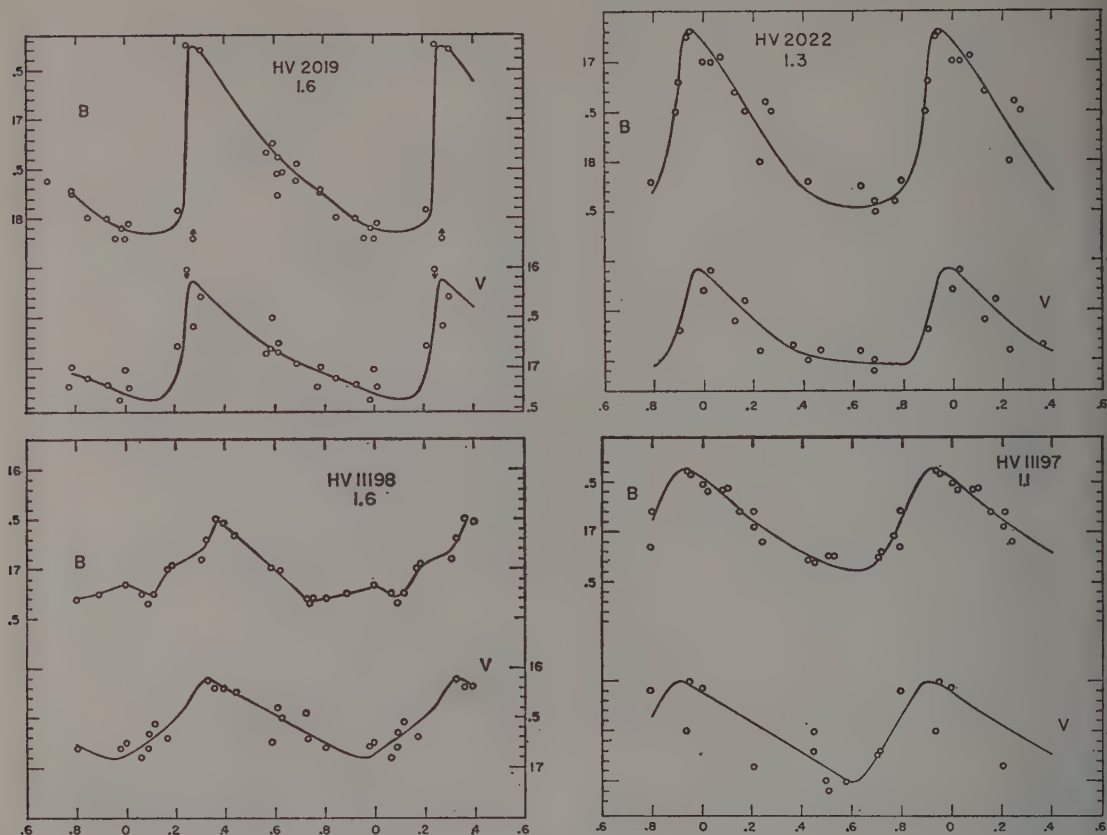


FIG. 7. Light curves for cepheids in the SMC.

maximum. In such cases both rates-of-rise are given in parentheses and when plotted a simple mean of the two has been used.

Of course, "bumps" can upset any one of the three light-curve parameters: amplitude, asymmetry, and rate-of-rise. It is not common, however, for all three parameters to be disturbed at the same time. The

light curve parameters used throughout the remainder of the discussion will be from the *B* light curves because the *B* light curves have large amplitudes and are generally better defined.

11. Crowding, Doubling, and Uncertain Periods

At the end of Table V a note refers to HV 1905. Around this variable, on the *B* plates, were companion stars which interfered with the measurement of the cepheid. Particularly at minimum light the companions caused the variable to read slightly brighter. But the effect of the companions is a strong function of the seeing, so that random as well as systematic errors were introduced. It should be stressed that the presence of the companions was quite noticeable but the effect on the light curve and derived parameter was not overly large. We conclude that if the companions were not enough less conspicuous to escape notice, that their effect on the light curve would be negligible.

HV 1898 is apparently a true, unresolved double. If we postulate an unresolved companion of roughly the same brightness and slightly bluer (one of the possible kinds of physical companions), then we explain in a roughly quantitative way: (1) the reduced amplitude

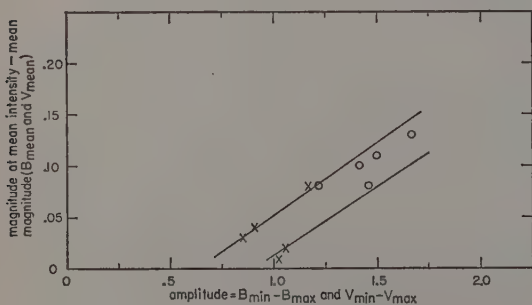


FIG. 8. The correction by which the mean magnitudes should be brightened when obtaining magnitudes at mean intensity. The circles represent the *B* light curves which have generally larger amplitudes than the *V* light curves represented by crosses. The scatter within the band is caused by light curves at the same amplitude not having the same shape.

TABLE V. Light curve parameters.

Variable HV	<i>P</i>	max	<i>B</i> min	mean*	max	<i>V</i> min	mean* Note	Asym- metry (<i>B</i>)	Rise (<i>B</i>)
1877	49.5	13.40	(14.50)	13.97	(12.67)	13.47	13.10	(0.74) <i>b</i> †	0.19
837	42.6	13.24	14.76	14.08	12.66	13.57	13.11	0.82	0.11
11182	39.6	13.89	15.11	14.60	13.22	13.82	13.60	0.72	0.19
2064	33.7	13.90	15.20	14.65	13.22	14.12	13.69	0.80	0.09
840	33.1	13.47	15.14	14.42	12.94	14.04	13.45	0.86	0.07
1967	29.1	13.63	(14.80)	14.29	13.02	13.91	13.51	0.82	0.12
847	27.2	13.87	15.29	14.76	13.36	14.21	13.85	(0.70)	0.08
1884	18.11	14.48	15.71	15.25	13.81	14.62	14.30	0.62 <i>b</i>	(0.12)
1925	17.18	13.76	14.92	14.41	13.34	14.06	13.83	(0.64)	0.26
1954	16.71	13.69	15.00	14.49	13.30	14.07	13.82	0.56	(0.44), (0.16)
1787	16.22	14.20	15.62	15.07	13.75	14.65	14.22	0.63 <i>b</i>	(0.11)
843	14.70	15.00	16.53	15.81	14.32	15.37	14.84	0.64 <i>b</i>	(0.10)
1695	14.50	14.63	15.77	15.23	14.03	14.72	14.41	0.74	0.14
1933	13.76	14.12	15.31	14.87	(13.72)	14.43	14.23	(0.64)	0.25
1873	12.91	14.77	15.86	15.41	14.17	15.00	14.72	0.65	0.14
1744	12.59	14.15	15.42	15.04	13.84	14.77	14.35	(0.55) <i>b</i>	0.26
856	12.13	14.82	15.98	15.47	14.23	14.99	14.76	0.74	0.17
2017	11.43	14.72	15.85	15.30	14.05	14.78	14.51	0.54 <i>b</i>	(0.19)
2063	11.18	14.90	15.92	15.45	14.08	14.90	14.59	0.61 <i>b</i>	0.22
2060	10.21	14.43	15.39	14.97	13.83	14.74	14.31	0.63	0.34
1768	9.798	15.36	15.89	15.60	14.56	15.13	14.84	0.58	0.56
1950	7.980	15.37	16.16	15.74	14.52	15.09	14.81	0.59	0.48
1905	7.413	15.01	15.94	15.51	14.53	15.33	14.93	†	0.13
1855	6.830	15.56	16.44	15.96	14.85	15.64	15.18	0.66	0.27
1945	6.465	15.29	16.52	15.92	14.73	15.61	15.19	0.80 <i>b</i>	0.13
11193	6.440	16.09	16.78	16.40	15.16	15.95	15.53	0.57	0.48
1858	6.120	15.69	16.64	16.18	15.02	15.65	15.44	0.78	0.15
1892	5.660	15.90	16.63	16.26	15.20	15.71	15.45	0.70	0.34
1929	5.579	15.67	16.23	16.03	15.03	15.62	15.30	0.71	0.39
1903	5.091	15.47	16.82	16.22	15.10	15.84	15.55	0.81	0.11
1827	4.924	15.27	16.53	16.05	15.00	15.83	15.44	0.82	0.10
1934	4.885	15.67	16.74	16.25	15.02	15.84	15.51	0.83	0.09
1785	4.725	15.85	16.82	16.53	15.21	16.09	15.73	0.81	0.14
1966	4.270	15.02	16.48	15.95	14.70	15.75	15.36	0.82	0.11
1825	4.265	15.40	16.62	16.19	15.07	15.97	15.61	0.84	0.06
1994	4.217	15.64	17.00	16.47	15.28	16.25	15.78	0.81	0.11
1793	4.180	15.79	17.13	16.51	15.27	16.30	15.80	0.74	0.14
1891	3.451	15.62	17.15	16.53	15.20	16.47	15.84	0.84	0.08
11206	3.400	16.13	17.20	16.61	15.53	16.19	15.98	0.75	0.14
1987	3.130	15.74	17.13	16.64	15.33	16.29	15.98	0.78	0.06
1898	3.021	15.79	16.53	16.29	15.47	16.14	15.90	†	0.11
2027	2.984	16.10	17.64	16.87	15.39	16.47	16.11	0.78	0.09
1974	2.896	15.75	17.32	16.82	15.52	16.45	16.05	(0.88) <i>b</i>	0.05
2000	2.880	15.87	17.23	16.76	15.59	16.52	16.13	0.75	0.13
1809	2.825	15.83	17.29	16.85	15.43	16.55	16.12	(0.81)	0.08
2049	2.800	16.12	17.30	16.83	15.59	16.63	16.14	0.89	0.08
1850	2.760	15.61	17.16	16.48	15.27	16.28	15.90	0.77	0.12
1981	2.724	16.27	17.80	17.25	15.88	17.04	16.49	0.82	0.07
1923	2.564	15.94	17.52	16.90	15.62	16.62	16.28	0.82	0.08
2046	2.554	16.01	17.35	16.76	15.54	16.45	16.04	0.71	0.19
850	2.5350	15.75	17.42	16.85	15.40	16.57	16.20	0.90	0.02
2021	2.4875	16.17	17.70	17.16	15.89	17.08	16.67	0.83	0.05
11192	2.3543	15.76	17.63	16.96	15.59	16.80	16.27	0.89	0.08
2002	2.345	16.02	17.43	16.78	15.54	16.60	16.24	0.84	0.08
844	2.219	15.80	17.46	16.94	15.73	16.66	16.31	0.86	0.06
2014	2.2028	15.83	17.47	16.86	15.33	16.50	16.20	0.84	0.07
848	2.178	16.22	17.68	16.99	(15.83)	(16.85)	16.35	0.63	0.23
11199	2.0952	16.15	16.75	16.50	15.69	16.22	15.92	0.80	0.29
2035	1.9800	16.42	17.79	17.20	16.14	17.11	16.72	0.60	0.13
Arp a	1.6568	16.73	(18.30)	17.85	16.42	(17.37)	17.00	(0.66)	0.08

TABLE V.—Continued

Variable HV	P	max	B min	mean*	max	V min	mean* Note	Asym- metry (B)	Rise (B)
2019	1.6393	16.26	18.13	17.49	16.12	17.33	16.91	0.75	0.02
11190	1.6180	16.51	17.30	17.05	16.12	16.92	16.57	(0.60)	0.37
2022	1.3090	16.69	(18.46)	17.92	16.57	17.55	17.28	(0.56)	0.07
11197	1.0736	16.38	17.40	16.94	(16.02)	(17.00)	16.52	0.70	0.20
Arp b	(2.078)	16.74	17.67	17.18	16.25	(17.1)	16.65	†	0.80
1978	(2.7)	16.2	17.2	16.7	15.4	16.0	15.8	†	
1868	(2.8)	16.3	17.0	16.75	16.5	17.1	16.8	†	
1836	(2.9)	16.0	16.9	16.5	15.6	16.3	16.0	†	
Arp g	(4-5)	15.1	15.6	15.35	15.0	15.5	15.3	†	

* This is mean of magnitude curve. See Table VII for magnitude at mean intensity ($\langle B \rangle$).
† Small "B" denotes bump on light curve.
‡ HV 1905. Optical companions, principally in B, account for scatter of measures and slightly blue mean color.
HV 1898. This cepheid, from its reduced amplitude and blue mean color, evidently has an unresolved blue companion of roughly the same brightness.
Arp b. No period can be found which satisfied all observations. The period 2.078 works well through the first 40 cycles, but is only approximately right thereafter.
HV 1978. Variable is small amplitude, too bright and too red. Conclude unresolved companion is red.
HV 1868. Optical companions introduce too much scatter.
HV 1836. Same as above.
Arp g. Period is around 4-5 days but amplitude is very small. Conclude that the cepheid has unresolved B star companion of about same magnitude.

(2) the blue mean color, and (3) the excessive brightness for its period. Most compelling, however, is just the shape of the light curve. The light curve of HV 1898 very obviously has only a maximum which appears normal as it sticks out above the constant light of its companion. This is an important star because it gives us confidence that we can recognize from the shape of the light curve alone any variables which are seriously affected by unresolved companions.

The last five cepheids in Tables III, V, and VI all have uncertain periods. The first of these, Arp b, appears to be a normal cepheid except that either its correct period has not been found or it has a variable period. The third and fourth variables of this group have optical companions which reduce the amplitude and introduce scatter so that the correct period is difficult to find. The second and fifth variables, HV 1978 and Arp g, apparently have unresolved companions. Again the reduced amplitude probably accounts for the difficulty in deriving its period.

All these stars are carried throughout the following analyses to furnish control, to furnish an extreme example of each one of the above three errors.

12. Accuracy of Parameters and Sources of Error

The following is an estimate of the errors in this study which might give spurious differences for two identical cepheids:

(1) Accidental error of single cepheid measure: ± 0.1 mag. Of, course, the error of the light curve at any point is reduced by the number of points defining that portion. Moreover, working with up to 27 points on the whole light curve and then integrating over the area of the light curve to obtain $\langle B \rangle$ and $\langle V \rangle$ reduces the error from the above source to the order of ± 0.01 mag. for mean quantities.

(2) Process of obtaining $\langle B \rangle$ $\langle V \rangle$ from light curves: ± 0.01 mag. Since these two sources of error are so small, a third source of error becomes important.

(3) Systematic background differences from area to area: ± 0.02 . This quantity is difficult to estimate but, as discussed at the end of Sec. 7, the iterative procedure made corrections which, averaged over corrected and uncorrected plates, were less than 0.1 mag. It is estimated then that the subregions are accurate with respect to each other to within the above figure.

For two cepheids of about the same brightness the over-all estimated relative error in their $\langle B \rangle$ $\langle V \rangle$ is about ± 0.03 mag.

There are other sources of error which undoubtedly occur for certain variables:

- (4) Crowding by companions: $< \pm 0.1$ mag.
- (5) Unresolved doubles: $< \pm 0.1$ mag.
- (6) Incorrect periods: ?

These errors probably occur for so few of the cepheids however, that they do not change the average estimated uncertainty of about ± 0.03 mag. just made.

When comparing cepheids of much different brightness in this study, we must take into account:

(7) Systematic accuracy of magnitude and color scales: ± 0.05 mag.

TABLE VI. Comparison of magnitude at mean light intensity to mean magnitude.

Variable HV	Magnitude mean B	Magnitude mean V	Intensity mean B	Intensity mean V
847	14.76	13.85	14.66	13.82
1966	15.95	15.36	15.84	15.34
1825	16.19	15.61	16.11	15.57
848	16.99	16.35	16.91	16.34
850	16.85	16.20	16.72	16.12

TABLE VII

PARAMETERS DERIVED FROM LIGHT CURVES

Variable HV	log P		<V>	-<V>	Δ	Δ(-<V>) [†]	Variable HV	log P		<V>	-<V>	Δ	Δ(-<V>)
1877	1.695	13.93	13.10	0.83	+0.01	-0.03	1994	0.625	16.39	15.75	0.64	+0.07	-0.02
837	1.629	13.88	13.09	.89	-.09	+ .01	1793	0.621	16.43	15.76	.67	+ .11	.00
11182	1.598	14.54	13.60	.94	+ .40	+ .08	1891	0.538	16.43	15.77	.66	-.07	.00
2064	1.528	14.58	13.67	.91	+ .29	+ .07	11208	0.532	16.57	15.98	.59	+ .05	.03
840	1.520	14.30	13.40	.90	-.01	+ .04	1987	0.496	16.55	15.95	.60	-.05	-.03
1987	1.464	14.23	13.49	.74	-.20	-.09	1898	0.480	16.29	15.90	.39	-.34*	-.18*
847	1.435	14.87	13.84	.83	+ .17	+ .01	2027	0.475	16.76	16.08	.70	+ .12	+ .06
1884	1.258	15.19	14.29	.90	+ .29	+ .09	1974	0.462	16.71	16.03	.68	+ .04	+ .04
1925	1.235	14.36	13.83	.53	-.59	-.24	2000	0.459	16.66	16.11	.55	-.02	-.05
1954	1.223	14.41	13.82	.59	-.57	-.19	1809	0.451	16.76	16.07	.69	+ .07	+ .05
1787	1.210	14.98	14.20	.78	-.02	-.01	2049	0.447	16.77	16.10	.67	+ .06	+ .04
843	1.187	15.70	14.78	.92	+ .60	+ .14	1850	0.441	16.37	15.87	.50	-.35	-.11
1895	1.161	15.18	14.41	.77	+ .06	0	1981	0.435	17.15	16.44	.71	+ .42	+ .11
1933	1.139	14.81	14.23	.58	-.96	-.17	1923	0.409	16.79	16.25	.54	0	-.03
1873	1.111	15.35	14.71	.64	+ .12	-.09	2046	0.407	16.68	16.02	.66	-.11	+ .02
1744	1.100	14.97	14.33	.64	-.28	-.12	850	0.404	16.73	16.14	.59	-.07	-.01
856	1.084	15.42	14.76	.66	+ .14	-.07	2021	0.398	17.06	16.61	.45	+ .24	-.05
2017	1.058	15.25	14.51	.74	-.10	-.02	11192	0.372	16.81	16.21	.60	-.06	+ .01
2063	1.048	15.41	14.58	.83	+ .03	+ .05	2002	0.370	16.69	16.20	.49	-.18	-.07
2060	1.009	14.94	14.29	.65	-.52	-.11	844	0.346	16.82	16.28	.54	-.10	-.03
1768	0.991	15.60	14.84	.76	+ .10	+ .01	2014	0.343	16.74	16.14	.60	-.19	.00
1950	0.902	15.73	14.81	.92	+ .05	+ .14	848	0.338	16.90	16.31	.59	-.04	+ .01
1905	0.870	15.49	14.92	.57	-.28*	-.14*	11199	0.321	16.50	15.92	.58	-.48	-.04
1855	0.834	15.94	15.17	.77	+ .09	+ .04	2035	0.297	17.12	16.69	.43	+ .08	-.05
1945	0.811	15.86	15.17	.69	-.04	-.03	Arp a	0.219	17.74	16.97	.77	+ .53	+ .20
11193	0.809	16.40	15.53	.87	+ .50	+ .14	2019	0.212	17.34	16.85	.49	+ .11	+ .01
1858	0.787	16.15	15.44	.71	+ .19	+ .01	11198	0.209	17.04	16.56	.48	-.20	-.04
1892	0.753	16.26	15.45	.81	+ .23	+ .09	2022	0.117	17.79	17.25	.54	+ .36	+ .09
1929	0.747	16.03	15.30	.73	-.01	+ .02	11197	0.031	16.91	16.49	.42	-.72	-.08
1903	0.707	16.14	15.55	.59	+ .01	-.07	Arp b	(0.318)	17.16	16.64	.52	+ .17	.00
1827	0.692	15.98	15.43	.55	-.19	-.11	1978	0.43	16.7	15.8	.9	-.05*	+ .18
1934	0.689	16.21	15.50	.71	+ .03	+ .01	1868	0.45	16.75	16.8	-.05	+ .05*	-.1
1785	0.674	16.50	15.71	.79	+ .29	+ .09	1836	0.46	16.5	16.0	.5	-.2	-.10
1968	0.630	15.85	15.32	.53	-.45	-.14	Arp g	.6-.7	15.35	15.3	.05	-1	...
1825	0.630	16.13	15.59	.54	-.17	-.10							

*Designates note at end of Table V.

[†]Measured along reddening line: Δ<V> = 3.0 Δ(-<V>).

Finally, differences which are not intrinsic can appear due to:

(8) Back-to-front magnitude differences. The difference in magnitude between variables at the front of system of angular radius α and the back is:

$$\Delta m = 5 \log[(1 + \tan \alpha)/(1 - \tan \alpha)]. \quad (7)$$

With an extreme optical diameter of 4° for the SMC, the back-to-front magnitude difference is only 0.16 mag. For the region investigated here, where the majority of variables are probably within a sphere of 1° in diameter, the total range is only 0.04 mag. and the correction to the mean is 0.02 mag., at most.

(9) Reddening and obscuration. This last effect will be discussed in a later section.

To summarize, the total estimated average error for cepheids at the same brightness is about ± 0.03 mag. in magnitude and color. For cepheids over the range of brightness here the average relative accuracy is about ± 0.05 mag., except for the faintest variables where the errors undoubtedly increase somewhat.

13. The Measured Period-Luminosity Relation

Table VII lists the and <V> derived for each cepheid investigated here. The is plotted against log P in Fig. 9. A mean straight line has been passed through the points in Fig. 9 by eye estimate. The equation of that line is

$$\begin{aligned} = 17.70 - 2.23 \log P \quad \text{observed P-L relation} \\ \pm 0.10 \quad \pm 0.10 \quad \text{estimated uncertainty.} \end{aligned} \quad (8)$$

It should be noted that this P-L relation is slightly different from that of a preliminary announcement (Arp 1958 a). The present P-L relation is 0.1 mag. brighter in zero point. This simply represents the average brightening of the cepheids when intensity means are used in place of the original magnitude means.

The scatter about the P-L relation drawn in Fig. 9 is the same as announced earlier (Arp 1960), an extreme range of about $\Delta = 1.1$ mag. This dispersion is roughly the same as in P-L relations derived by previous investigators (Shapley 1930; 1940).

14. Observed Place of the Cepheids in the Color-Magnitude Diagrams

Figure 10 shows a plot of <V> vs -<V>. It seems that the cepheids, at mean light, appear only in a region between 0.3 and 0.4 mag. wide in color index which runs roughly from -<V> = 0.9 mag. for the brightest to -<V> = 0.4 mag. for the faintest. If fitted with a straight line, the points in Fig. 10 are best represented by

$$\begin{aligned} -<V> = 2.2 - 0.1 <V> \\ \text{apparent domain of cepheids} \\ \text{in color-magnitude diagram.} \end{aligned} \quad (9)$$

This is the same equation quoted previously (Arp 1958 a). For the best fit of the points in the diagram, however, is necessary to draw a break in slope of the mean color-magnitude line, as shown in Fig. 10. Below about <V> = 15.5 mag. the cepheids get bluer faster with decreasing luminosity than do the brighter cepheids.

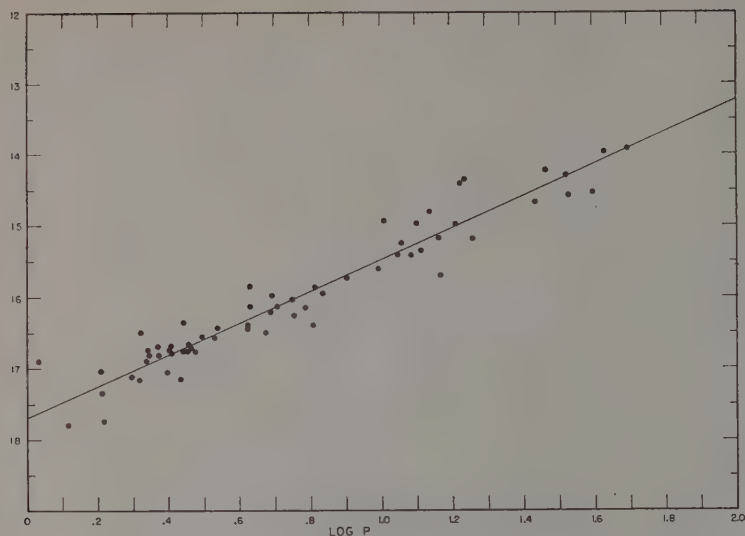


FIG. 9. The period-luminosity relation. These are magnitudes at mean light as measured, from columns 3 and 4 of Table VI. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

ANALYSIS OF THE DATA

Studies of the richly populated and relatively unreddened globular clusters have shown that RR Lyrae cepheids have colors which range through $\Delta(B-V) = 0.2$ mag. at the same luminosity. By analogy, longer period cepheids would be expected to occur in a strip in the color-magnitude diagram which had a width of this order. This expectation has been supported by recent observations of star clusters in the SMC which have rich enough giant branches to define the instability gap. In NGC 458 (Arp 1958 a) the gap was 0.3 mag. in $B-V$; in NGC 330, a brighter SMC cluster,

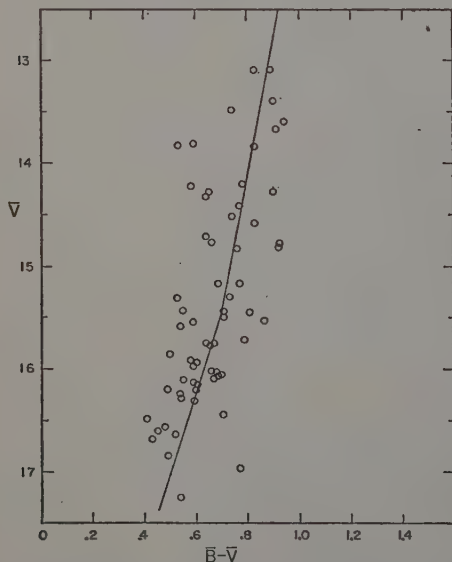


FIG. 10. Apparent place of the cepheids at mean light in the color-magnitude diagram. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

the gap was greater than 0.4 mag. in $B-V$ (Arp 1959 b). See Fig. 28.

More direct evidence comes from a plot of the cepheids themselves in the color-magnitude diagram. Figure 10 shows that, as observed, the dispersion in color index is more than 0.3 mag. at a given luminosity. Since it will be shown later that the reddening for these cepheids is probably very small, the intrinsic dispersion in color for these long-period cepheids is of just the order of $B-V = 0.3$ mag.

A characteristic period, from the general $P\sqrt{\rho} = \text{const}$ law for vibrating systems, can be associated with every point within this strip of instability. The lines of constant period slant toward redder colors and fainter luminosities. See Fig. 25. Therefore, a cepheid situated on the blue side of the gap finds its characteristic period shorter than cepheids of the same luminosity in the center of the gap. This particular cepheid would fall above the mean $P-L$ relation. Since the whole strip is more or less filled with cepheids, in this way there will be caused a real dispersion in the $P-L$ relation. (In fact, there will be no unique relationship between the period and any other single parameter, such as color or amplitude.) The sense of this dispersion will be that at a given period the bluest cepheids are the brightest and the faintest cepheids are the reddest. This is the same effect which interstellar reddening would give. In fact, we shall see that the ratio of intrinsic faintness to redness is so close to the reddening to absorption ratio that it is essentially impossible to use the observed colors in conjunction with the observed magnitudes to derive reddening corrections for a group of cepheids, even if they are all at the same distance.

This point is important because, for example, if we look at the observed $P-L$ relation of Fig. 9, we cannot say by using the observations of the colors of the

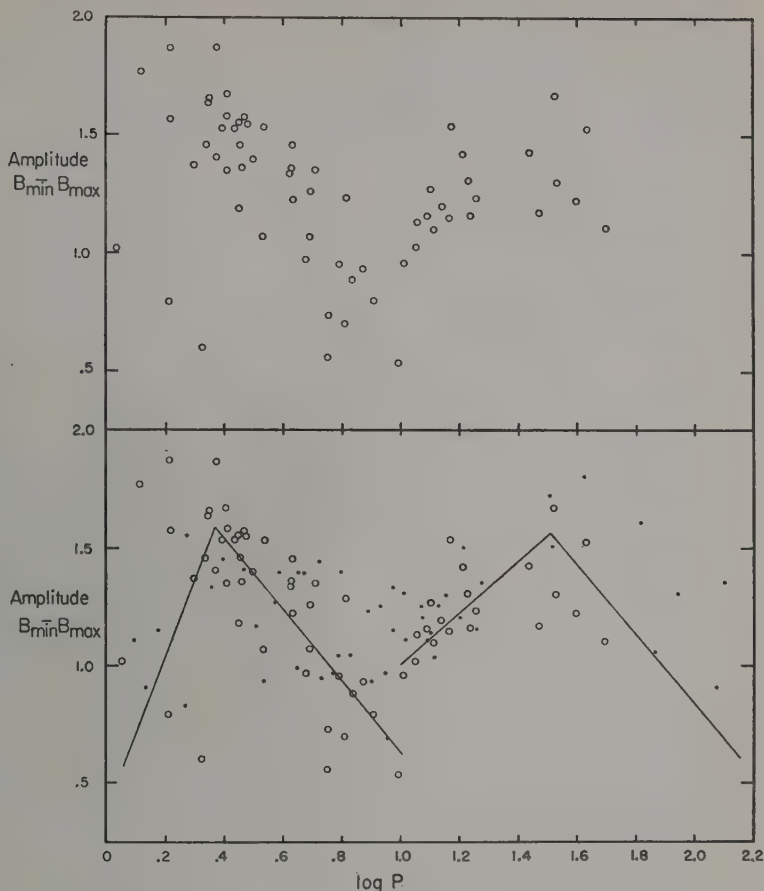


FIG. 11. (Above) The period-amplitude relation for 63 cepheids observed here.

FIG. 13. (Below) The period-amplitude relation for 63 cepheids observed here (large circles), plus cepheids reported by Shapley and McKibben (small circles) and Gascoigne (small circles), and one 9-day LMC cepheid reported by Apachkin (small circle). The straight line shows the adopted mean period-amplitude relation.

stars how much of the dispersion in the $P-L$ relation is intrinsic and how much is due to variable absorption. If we wish to study the intrinsic nature of the cepheids, when it is necessary to search for some other method of removing reddening.

We attempt to do this here by considering stars only of a given period, that is, stars which lie along a given $P=\text{const}$ line in the color-magnitude diagram. Since differences in color and brightness could be caused by reddening, we cannot tell where in the gap, along that $P=\text{const}$ line, the star really belongs. Only if there is some property of the star which depends only on its position in the gap and not upon reddening can we hope to disentangle the two effects. Of course, the light curve shape is the only such property of the cepheids here which has a chance of telling us where in the gap the cepheid really belongs. For example, it is reasonable to expect that the cepheids toward the center of the gap have maximum instability and, therefore, have the largest amplitude of light variation. This is, of course, observed in the RR Lyrae stars (Roberts and Sandage 1955) where the amplitude of light variation falls to zero on either side of the gap. We will also investigate other possible characteristics of the light curve

which can be measured quantitatively, such as asymmetry, to see whether an accurate enough relation can be established between light-curve properties and position in the cepheid gap along a constant period line. At any given period, however, we must first establish what the properties of the normal or average light curve are in order to measure deviations which can be correlated with relative position in the gap.

15. The Period-Amplitude Relation

Fig. 11 shows the period-amplitude relation for 63 of the cepheids observed here. It is clear that generally these cepheids increase in amplitude for periods both longer and shorter than about 8 days. In order to check the suggested decrease in amplitude for periods longer than 33 days and shorter than 2.5 days, however, more points were needed.

The additional cepheid amplitudes were drawn from the sources indicated in Table VIII. The data listed by Gascoigne (1958) are on the B, V system and were used as he presented them. Before using the Harvard data, the amplitudes were corrected for the difference in magnitude scales between the two investigations. This was accomplished by noting that among the 49

TABLE VIII. Amplitude and asymmetry data from supplementary sources.

SMC variable HV	$\log P$	Amplitude ($B_{\min} - B_{\max}$) Gascoigne ¹	Amplitude ($B_{\min} - B_{\max}$) Harvard ²	Asymmetry Harvard ²
821	2.104	1.35		
829	1.947	1.3		
834	1.866	1.05		
824	1.818	1.6		
837*	1.629	1.7		
2195	1.621	1.80		
855	1.518		1.5	0.79
823	1.504	1.85	1.6	0.86
817	1.276	1.40	1.3	0.61
1342	1.254	1.15	0.7	0.59
1333	1.211		1.5	0.67
1328	1.200	1.00	1.4	0.63
1335			1.3	0.61
827	1.129	1.52	1.0	0.61
1351	1.116		1.03	0.54
2052	1.099		1.1	0.54
1365	1.093		1.15	0.63
856*	1.084		1.07	0.66
857	1.078		1.2	0.53
1610	1.066		1.25	0.53
2017*	1.058		1.11	0.54
2063*	1.048		0.95	0.58
1426	1.018		1.11	0.56
818	1.014		1.31	0.70
2060*	1.007		1.0	0.53
1768*	0.991		0.47	0.57
1334	0.975		1.33	0.71
836	0.973		1.15	0.77
2103	0.953		0.69	0.67
1790	0.947		0.97	0.77
1338	0.928		1.25	0.70
816	0.899		0.93	0.71
2081	0.881		1.23	0.73
1855*	0.835		0.95	0.72
1400	0.822		1.04	0.68
1492	0.799	1.4	1.39	0.70
2124	0.782		1.04	0.72
815	0.762		0.97	0.77
1818	0.737		0.95	0.75
2031	0.720		1.44	0.71
851	0.671		1.39	0.77
1425	0.658		1.39	0.79
1619	0.641		0.99	0.83
858	0.583		1.39	0.80
2085	0.570		1.27	0.80
11206*	0.531		0.93	0.74
2053	0.509		1.17	0.84
1460	0.463		1.41	0.88
2076	0.397		1.45	0.84
2128	0.357		1.33	0.70
1446	0.271		1.55	0.77
2127	0.262		0.83	0.57
10364	0.176		1.55	0.66
1513	0.133		0.91	0.57
1505	0.096		1.11	0.64
LMC† HV 2477	2.075		0.9	0.48

* Cepheids common to present investigation. See Tables V, VII, and IX.

† Longest period cepheid in LMC. Shapley and McKibben (1940) and Gaposchkin (1955). (Error on scale of light curve in latter reference.)

¹ Private communication; also Gascoigne (1958).² Amplitudes empirically transformed from Shapley *et al.* (1947).

accurate Harvard light curves of SMC variables (Shapley and McKibben 1940) there are seven which are common to the present investigation. They are marked by asterisks in Table VIII. Plotting the values obtained in the present investigation against values read from the seven Harvard light curves enabled an empirical straight-line transformation to be made between the Harvard amplitudes and the present amplitudes. This is shown in Fig. 12.

The magnitude scale correction shown in Fig. 12 would predict about 0.23 mag. correction for a variable with the average amplitude of 1.25 mag. The points in Fig. 12 indicate, however, that the fainter the variable the greater correction factor must be applied to the Harvard amplitudes. The small circles represent comparison of the present amplitudes to the amplitudes listed in *Harvard Circ.* 439 and 444. The scatter is mostly in the direction of large corrections, implying that in many cases the maximum or minimum was not well observed; hence the amplitude was underestimated in the early work. The 49 light curves listed in *Harvard Ann.* 109, when systematically corrected to the present system, are however very accurate—as shown by the large circles. The spread around the line in Fig. 12 for the stars is only ± 0.05 mag. (standard deviation). This demonstrates that the error in each amplitude determination is less than ± 0.04 mag., and that the scatter observed in the period-amplitude relations of Figs. 12 and 13 is dispersion inherent in the cepheids themselves.

The relation in Fig. 12 also enables the amplitudes of the 49 Harvard cepheids to be transformed accurately to the present system. They are plotted together with eleven Gascoigne cepheids in Fig. 13. In Fig. 13 the mean adopted period-amplitude relation is indicated by a line drawn through the points. It is difficult to be certain of the behavior of the mean line at around eight days, but the shorter-period group appears to extend to small amplitudes below the longer-period group and not join continuously onto the periods greater than eight days. This same behavior is suggested in the asymmetry plot although, here again, the situation around eight days is very confused.

16. The Period-Asymmetry Relation

After the amplitude, the most characteristic over-all feature of the light curve is its asymmetry. Asymmetry is defined here as the fraction of its cycle during which the variable is decreasing in brightness. The parameter is probably related to the degree to which the oscillation is free or driven. This quantity, as measured for 65 of the cepheids investigated here, is listed in the next column of Table V.

Values of the asymmetry were read from the published light curves of the 49 Harvard SMC cepheids (Shapley and McKibben 1940) and listed in the last column of Table VIII. The Harvard values should be directly comparable to the ones here since the asymmetry derived in this way is not dependent on magni-

AMPLITUDE ARP-
AMPLITUDE HARVARD

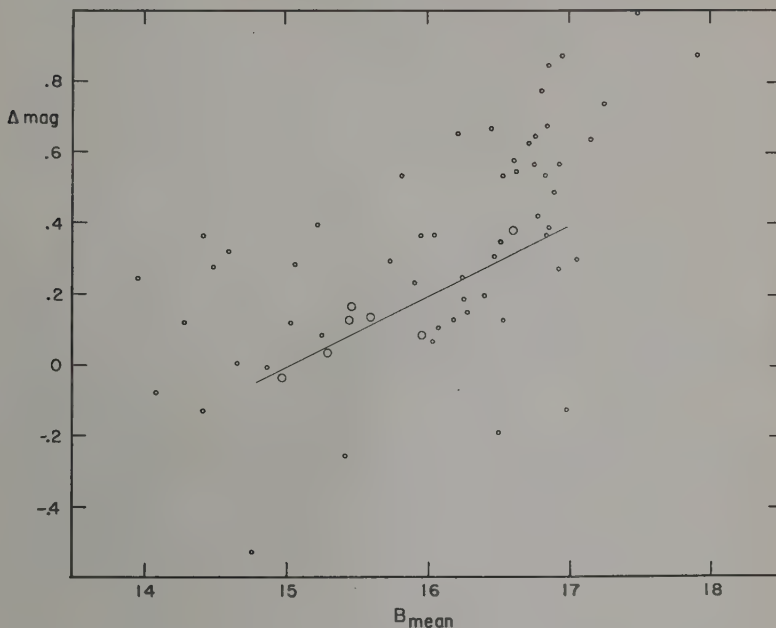


FIG. 12. The corrections to be applied to Harvard amplitudes to obtain amplitudes on the present system. Small circles are amplitudes from *Harvard Circ. 439* and *Harvard Ann. 109*, No. 10. The large circles are from the accurate light curves in *Harvard Ann. 109*, No. 10.

the scales. The seven stars common to the two investigations were compared. One, HV 2060, has a bump at maximum in the Harvard light curve which causes its asymmetry to be about 0.53, compared to the 0.3 derived here. Comparison of the remaining six, however, shows a standard deviation of only ± 0.04 . We conclude that the asymmetry is a quantity which characterizes the light-curve shape and which can be reliably measured to an accuracy of $\pm 3\%$.

The plot of asymmetry against $\log P$ is shown in Figure 14. The adopted mean relationship is drawn through the points. The over-all dispersion of the plotted points from the mean line is ± 0.06 . Only ± 0.03 of this is observational error, so,

$$[(0.03)^2 + x^2]^{\frac{1}{2}} = 0.06 \\ x = \pm 0.05.$$

The intrinsic dispersion from the mean $\log P$ -asymmetry relation is ± 0.05 .

The period-asymmetry relation resembles the period-amplitude relation very closely. The occurrences of maximum asymmetry are shifted to slightly longer periods than the points of maximum amplitude. At 2.5 days the shift is 0.05 in $\log P$, and at 33 days it is 0.02 in $\log P$. The point of minimum asymmetry, however, occurs at a period around 13 days, which is considerably longer than the period of 8 days at which minimum amplitude occurs.

17. The Period-Rate-of-Rise Relation

In the process of deriving the parameter of asymmetry, some light curves were encountered which had

very flat minima or bumps at minimum. In order to escape this uncertainty in the time of minimum and still obtain a light-curve parameter which was related to the asymmetry, the rate-of-rise of the light curve was measured. This quantity is listed in the last column of Table V. The slope of the ascending branch of the light curve was measured in units of the number of phase units required to rise one magnitude. Of course the measurement of this quantity is disturbed by bumps on the ascending branch or discrete changes in slope, such as in the case of HV 1884 and HV 1954. The effects of bumps or slope changes were removed as best possible by trying to estimate the slope of the underlying light curve with particular weight being given to the top of the ascending branch. Uncertain cases are enclosed by parentheses in Table V.

Asymmetry

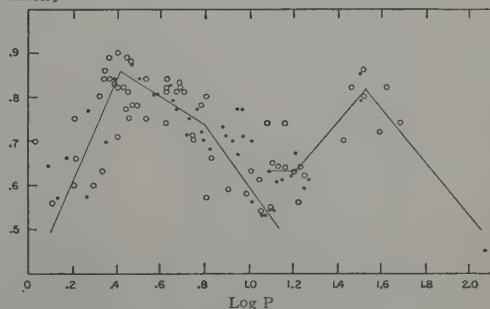


FIG. 14. The period-asymmetry relation. The open circles represent values for the present cepheids taken from column 9 of Table V. The small closed circles represent values for 49 cepheids published by Shapely and McKibben.

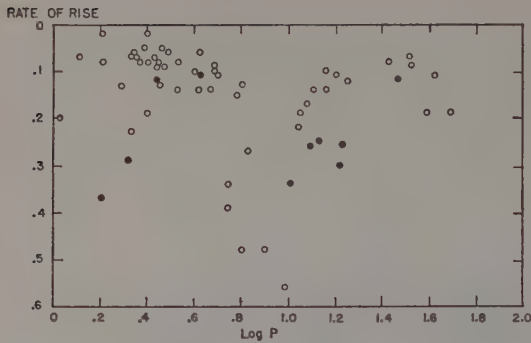


FIG. 15. The period-rate-of-rise relation. The rates-of-rise are taken from column 10 of Table V and the closed circles represent only stars which are more than 0.2 mag. brighter than the mean $P-L$ relation ($\Delta\langle B \rangle \leq -0.20$ mag.).

The relation between the rate-of-rise and $\log P$ for the variables here is shown in Fig. 15. The form of the relationship is somewhat different than in the period-amplitude plot and has more scatter although the maxima and minima are at about the same $\log P$. The period-rate-of-rise turns out to have one surprising feature, however, which is of great importance to the present analysis. In Fig. 15 all the stars which are 0.20 mag. or more brighter than the mean period luminosity relation ($\Delta\langle B \rangle \leq -0.20$ mag.) are designated by filled circles. These 11 stars have systematically slower rates-of-rise than other stars at the same period. Why this difference appears this way in the period-rate-of-rise relation and differently in the period-amplitude and period-asymmetry plot is a mystery. The important fact is that there is something demonstrably different about the light curves of these bluest-brightest cepheids compared to the remainder of the cepheids. The conclusion here must be, then, that these 11 variables, which are not only brighter than average but also much bluer than average, are not simply relatively unreddened in comparison to the average. The objective difference in their light curves shown in the period-rate-of-rise relation proves that these cepheids are *intrinsically* bluer and brighter. At the same time the fact that these stars are differentiated in the period-rate-of-rise plot and not the period-amplitude plot rules out the possibility that these 11 stars have unresolved blue companions which increase their brightness and blueness. If that were the case, of course, their amplitudes would be much reduced but their rate-of-rise to maximum brightness would be little affected.

Because the intrinsic properties are still discernible, the results imply strongly that the reddening suffered by this group of cepheids is not large compared to its dispersion in intrinsic properties. If this is the case, then there is the hope that a quantitative relationship can be established between the parameters of light-curve shape and deviation from average brightness.

18. Deriving the Reddening of the Cepheids

Figure 16 shows a plot of each cepheid's deviation from the mean period-amplitude relation, $\Delta(B_{\min} - B_{\max})$ versus its deviation from the mean period-apparent magnitude relation, $\Delta\langle B \rangle$. As outlined previously we would expect stars which were brighter than average (negative $\Delta\langle B \rangle$) to have small amplitudes and the smallest deviation from mean brightness. Stars on the red side of the gap would have small amplitudes again but would appear underluminous for their period. In short, we would expect a relative amplitude brightness relation of the general form of the solid line in Fig. 16.

Now, if variable absorption is present, each cepheid which suffers absorption will be moved horizontally away from the intrinsic line in Fig. 16. If the absorption is not too large, the intrinsic line will not be shredded out beyond recognition. The plotted points in Fig. 16 definitely do show a general relationship between the relative amplitude and brightness of the cepheids. We note that the cepheids with large relative amplitudes have only a small range in relative brightness. As we look at the cepheids with smaller amplitudes, the majority of them are found to be fainter than average. A few of the cepheids, however, are found to become brighter with diminishing amplitudes. These latter cepheids are just the cepheids demonstrated in Sec. 17 to have the light curve anomaly of slow rate-of-rise. They were thus demonstrated to be intrinsically bluer and would therefore be expected in Fig. 16 to have smaller amplitudes because they fall on the blue side of the cepheid gap.

We assume that most of the points in Fig. 16 have been moved horizontally away from some narrow, intrinsic line by the effects of absorption. It is difficult to be sure exactly where the unabsorbed line really is. The solid line in Fig. 16 has been drawn with the shape of the average of all the plotted points, and then shifted

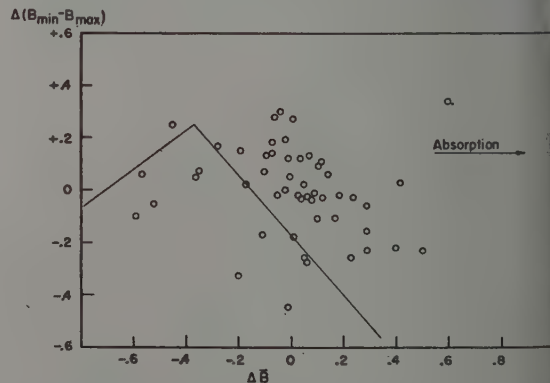


FIG. 16. The relative amplitude-relative luminosity relation. The ordinate is the difference in B amplitude between a given cepheid and the average amplitude at that period and taken from Table IX. The abscissa represents the amount, in magnitudes, which a given cepheid is brighter or fainter than the average at that period. (Also from Table X.) The effect of absorption on any cepheid is to move it horizontally toward fainter magnitudes from the unreddened line, $\bar{B} = \langle B \rangle$.

TABLE IX
 VARIATIONS FROM AVERAGE LUMINOSITY, AMPLITUDE, RATE-OF-RISE, AND ASYMMETRY

Variable HV	log P	$V_{\min} - V_{\max}$	$\Delta(V_{\min} - V_{\max})$	$\Delta\langle V \rangle_{P-L}$	$B_{\min} - B_{\max}$	$\Delta(B_{\min} - B_{\max})$	$\Delta\langle B \rangle_{P-L}$	Rate-of-rise (from B light curves)	Asymmetry
1877	1.895	(0.80)	+0.08	0.00	(1.10)	(-0.18)	+0.01	+0.03	+0.02
837	1.629	0.81	+0.06	-0.17	1.52	+0.13	-0.09	+0.00	+0.06
11182	1.598	0.60	-0.26	+0.26	1.22	-0.22	+0.40	+0.09	-0.06
2064	1.528	0.90	-0.06	+0.17	1.30	-0.23	+0.29	+0.01	-0.02
840	1.520	1.10	+0.13	-0.13	1.67	+0.12	-0.01	-0.01	+0.05
1967	1.464	0.89	-0.08	-0.16	(1.17):	(-0.33):	-0.20	+0.06	+0.04
847	1.435	0.85	-0.12	+0.11	1.42	-0.11	+0.17	+0.02	-0.06
1864	1.258	0.81	-0.05	+0.14	1.23	-0.06	+0.29	+0.04	-0.04
1925	1.235	0.72	-0.13	-0.38	1.16	-0.10	-0.59	+0.16	-0.00
1954	1.223	0.77	-0.12	-0.42	1.31	+0.06	-0.57	+0.25	-0.08
1787	1.210	0.90	+0.07	-0.07	1.42	+0.19	-0.02	+0.02	-0.00
843	1.167	1.05	+0.24	+0.40	1.53	+0.34	-0.60	+0.00	+0.01
1695	1.161	0.69	-0.12	+0.03	1.14	-0.03	+0.06	+0.03	+0.11
1933	1.139	0.71	-0.06	-0.22	1.19	+0.05	-0.36	+0.13	+0.01
1873	1.111	0.63	+0.06	+0.20	1.09	-0.03	+0.12	+0.01	+0.02
1744	1.100	0.93	+0.16	-0.21	1.27	+0.17	-0.28	+0.12	-0.06
856	1.084	0.76	-0.00	+0.18	1.16	+0.06	+0.14	+0.02	+0.11
2017	1.056	0.73	-0.01	-0.14	1.13	+0.07	-0.10	-0.00	-0.02
2063	1.048	0.82	+0.08	-0.08	1.02	-0.02	+0.03	-0.00	+0.05
2060	1.009	0.91	+0.19	-0.48	0.96	-0.05	-0.52	-0.00	+0.04
1768	0.991	0.57	-0.13	+0.04	0.53	-0.11	+0.10	-0.00	-0.02
1950	0.902	0.57	-0.08	-0.21	0.79	+0.02	+0.05	-0.00	-0.07
1905	0.870	0.80	+0.17	-0.18*	0.93	+0.09	-0.28*	-0.00	-0.00
1855	0.834	0.79	+0.19	-0.01	0.88	-0.01	+0.09	-0.00	-0.06
1945	0.811	0.88	+0.26	-0.06	1.23	+0.30	-0.04	-0.00	+0.07
11193	0.809	0.79	+0.16	+0.28	0.69	-0.23	+0.50	-0.00	-0.16
1858	0.787	0.63	-0.02	+0.13	0.95	-0.01	+0.19	-0.00	+0.04
1892	0.753	0.51	-0.17	-0.07	0.73	-0.26	+0.23	-0.00	-0.05
1928	0.747	0.59	-0.09	-0.10	0.56	(+0.45)†	-0.01	-0.00	-0.04
1903	0.707	0.74	+0.01	+0.05	1.35	+0.27	+0.01	+0.01	+0.04
1827	0.692	0.83	+0.09	-0.10	1.26	+0.15	-0.19	+0.01	+0.05
1934	0.689	0.82	+0.07	-0.05	1.07	-0.03	+0.03	-0.00	+0.06
1785	0.674	0.88	+0.10	+0.13	0.97	-0.16	+0.29	+0.06	+0.03
1966	0.630	1.05	+0.24	-0.37	1.46	+0.25	-0.45	+0.04	+0.03
1825	0.630	0.90	+0.09	-0.10	1.22	+0.02	-0.17	-0.01	+0.05
1994	0.625	0.97	+0.15	+0.06	1.36	+0.13	+0.07	+0.04	+0.02
1793	0.621	1.03	+0.21	-0.11	1.12	+0.11	+0.07	+0.02	+0.12
1891	0.538	1.27	+0.36	-0.04	1.53	+0.18	-0.07	+0.03	+0.03
11206	0.532	0.66	-0.25	+0.07	1.07	-0.26	+0.05	+0.09	-0.07
1987	0.496	0.96	+0.01	-0.06	1.39	-0.01	-0.05	+0.02	-0.05
1896	0.460	0.67	-0.30	-0.15	0.74	-0.68	-0.34*	-0.00	-0.00
2027	0.475	1.08	+0.11	-0.00	1.54	+0.11	+0.12	+0.05	-0.06
1974	0.462	0.93	-0.05	-0.38	1.57	+0.12	+0.04	+0.01	+0.04
2000	0.459	0.93	+0.05	-0.05	1.36	0	+0.02	-0.00	-0.00
1809	0.451	1.12	+0.12	-0.05	1.46	-0.02	+0.07	+0.04	-0.04
2049	0.447	1.04	+0.04	-0.03	1.18	-0.28	+0.06	+0.04	+0.04
1850	0.441	1.01	-0.00	-0.27	1.55	+0.07	-0.35	+0.04	-0.08
1981	0.435	1.16	+0.14	+0.28	1.53	+0.03	+0.42	+0.03	-0.03
1923	0.409	1.00	-0.03	+0.03	1.58	+0.05	0	+0.04	-0.03
2046	0.407	0.91	-0.13	-0.21	1.34	-0.17	-0.11	-0.00	-0.14

*Refer to Note † of Table V.

†May belong to extension of longer period line and be: $\Delta(B_{\min} - B_{\max}) = -0.16$.

that it passes through the points which show the least absorption. This means that we have assumed these stars, which represent about 10% of the whole group, to be unreddened. Now it is possible to shift each point in Fig. 16 back to the full line and read off the total absorption, $\Delta\langle B \rangle_{\text{abs}}$, which each star has presumably suffered in being moved from the original, full-line relationship. These derived absorption values are listed in the second column of Table X.

Now correlations of cepheid brightness with light-curve shape are also available for the light-curve parameters of asymmetry and rate-of-rise. The residuals from the mean rate-of-rise and asymmetry relations are

given in the last two columns of Table IX. The plots are in general similar to Fig. 16, since all these light-curve parameters are closely correlated. As would be expected from the way in which the bluest variables were outstanding in the period-rate-of-rise relation, the relative rate-of-rise versus relative brightness plot expands the scale for this blue, smaller amplitude group. The relative lengths of the two straight line portions of Fig. 16 are interchanged. There is more scatter in the last two plots of relative brightness versus relative rate-of-rise and asymmetry than in Fig. 16, and they are not shown. The values of absorption derived from these

TABLE IX—continued

Variable HV	log P	$V_{\min} - V_{\max}$	$\Delta(V_{\min} - V_{\max})$	$\Delta\langle V \rangle_{P-L}$	$B_{\min} - B_{\max}$	$\Delta(B_{\min} - B_{\max})$	$\Delta\langle B \rangle_{P-L}$	Rate-of-rise (from B light curves)	Asymmetry
850	0.404	1.17	+0.13	-0.10	1.67	+0.14	-0.07	-0.02	+0.05
2021	0.396	1.19	+0.13	+0.35	1.53	-0.03	+0.24	+0.01	-0.00
11192	0.372	1.21	+0.19	-0.11	1.87	+0.28	-0.06	+0.04	+0.08
2002	0.370	1.06	-0.03	-0.12	1.41	-0.18	+0.18	+0.04	+0.03
844	0.346	0.93	-0.11	-0.09	1.66	+0.15	-0.10	-0.00	+0.08
2014	0.343	1.17	+0.13	-0.24	1.64	+0.15	-0.19	+0.03	+0.05
848	0.338	(1.02)	-0.01	-0.09	1.46	-0.00	-0.04	-0.00	-0.13
11199	0.321	0.53	-0.49	-0.53	0.60	-0.53	-0.48	+0.24	+0.04
2035	0.297	0.97	-0.01	+0.19	1.37	+0.03	+0.08	+0.06	-0.12
Arp a	0.219	(0.95)	+0.08	+0.28	(1.57)	+0.49	+0.53	+0.04	+0.03
2019	0.212	1.21	+0.34	+0.15	1.87	+0.80	+0.11	+0.05	+0.13
11198	0.209	0.80	-0.06	-0.15	0.79	-0.27	-0.20	+0.25	-0.02
2022	0.117	0.98	+0.25	+0.32	1.77	+0.02	+0.36	+0.14	+0.06
11197	0.031	(0.98)	+0.35	-0.06	1.02	-0.00	-0.72	-0.00	-0.00
Arp b	(0.318)	(0.85)	-0.16	+0.19	(0.93)	-0.00	+0.17	-0.00	-0.00

TABLE X. Absorption estimated from magnitude light curve shape relations.

Variable HV	$\Delta\langle B \rangle_{\text{amp}}$	Absorption in B $\Delta\langle B \rangle_{\text{rate-of-rise}}$	$\Delta\langle B \rangle_{\text{asym}}$	Adopted absorption $\Delta\langle B \rangle_{\text{abs}}$	Intrinsic $\langle V \rangle_0$	$(\langle B \rangle - \langle V \rangle)$
1877	0.00	0.10	0.22	0.11	13.01	0.80
837	0.19	0.17	0.27	0.21	12.94	0.84
11182	0.36	0.17	0.31	0.28	13.39	0.87
2064	0.24	0.49	0.37	0.37	13.40	0.82
840	0.25	0.30	0.31	0.29	13.19	0.83
1967	...*	+0.25	+0.30	0.28	13.28	0.67
847	0.22	0.32	0.08	0.21	13.69	0.78
1884	0.39	0.33	0.28	0.33	14.05	0.82
1925	0.25	0.22	0.05	0.17	13.71	0.49
1954	0.05	0.35	0.33	0.24	13.64	0.53
1787	0.30	0.11	0.12	0.18	14.05	0.73
843	0.97	0.86	0.77	0.87	14.12	0.70
1695	0.18	0.15	(0.33)	0.17	14.29	0.73
1933	0.28	0.31	0.24	0.28	14.02	0.51
1873	0.24	0.33	0.33	0.30	14.47	0.56
1744	0.20	0.36	0.29	0.28	14.12	0.57
856	0.34	0.29	(0.41)	0.35	14.49	0.57
2017	0.12	0.16	-0.02	0.14	14.39	0.70
2063	0.16	...	0.35	0.25	14.40	0.77
2060	0.26	...	-0.01	0.26	14.08	0.58
1768	0.16	...	0.16	0.16	14.72	0.72
1950	0.22	...	(-0.07)	0.22	14.63	0.86
1905	0.31	...	(0.02)	0.31	14.68†	0.49†
1855	0.23	...	(-0.11)	0.23	14.99	0.71
1945	0.33	...	0.45	0.39	14.87	0.59
11193	0.55	...	0.04	0.30	15.29	0.79
1858	0.32	...	0.47	0.39	15.14	0.61
1892	0.16	...	0.11	0.14	15.33	0.77
1929	?	...	(-0.04)	0.24†	15.12	0.67
1903	0.38	0.22	0.30	0.30	15.31	0.51
1827	0.09	0.13	0.12	0.11	15.34	0.52
1934	0.16	0.29	0.29	0.28	15.29	0.64
1785	0.30	0.22	0.54	0.35	15.44	0.70
1966	-0.08	-0.05	0.08	-0.02	15.35	0.54
1825	0.00	0.09	0.15	0.08	15.53	0.52
1994	0.34	0.11	0.28	0.24	15.57	0.58
1793	0.36	0.00	0.06	0.14	15.64	0.63
1891	0.24	0.03	0.14	0.14	15.65	0.62
11206	-0.02	-0.18	-0.07	-0.09	16.04	0.61
1987	0.08	0.10	-0.10	0.03	15.92	0.59
1898	...	...	...	...	...	...
2027	0.37	0.11	0.33	0.17	15.94	0.66
1974	0.29	0.25	0.31	0.28	15.82	0.61
2000	0.13	...	...	0.13	16.02	0.52
1809	0.20	0.11	0.05	0.12	15.98	0.66
2049	-0.03	0.10	0.35	0.14	15.98	0.63
1850	0.26	0.17	0.51	0.31	15.63	0.42
1981	0.60	0.52	0.45	0.52	16.05	0.58
1923	0.20	0.04	0.05	0.10	16.16	0.51
2046	-0.11	...	(-0.52)	0.24†	15.84	0.60
850	0.20	0.26	0.27	0.24	15.96	0.53
2021	0.36	0.45	0.38	0.40	16.31	0.35
11192	0.30	0.34	0.31	0.31	15.97	0.52
2002	(-0.18)	0.22	0.12	0.05	16.17	0.48
844	0.18	0.25	0.27	0.23	16.10	0.48
2014	0.10	0.18	0.19	0.16	16.02	0.56
848	0.11	...	...	0.11	16.22	0.56
11199	...	0.51	-0.01	0.24†	15.74	0.52
2035	0.26	0.02	(-0.18)	0.24†	16.51	0.37
Arp a	0.90	0.73	0.78	0.80	16.37	0.57
20.9	0.48	0.08	0.31	0.29	16.64	0.42
11198	(-0.34)	(0.82)	(-0.10)	0.24†	16.38	0.42
2022	(0.63)	(-0.14)	(0.72)	0.24†	17.07	0.48
11197	...	...	...	0.24†	16.31	0.36
Arp b	...	...	...	0.24†	16.46	0.46

* May have missed min.

† Individual value uncertain—mean absorption adopted.

‡ Not plotted. See notes † Table V.

TABLE XI. Three-color measures of SMC supergiants.

Star	$B-V$	Gascoigne ¹ $U-B$	E_{B-V}	$B-V$	Wesselink ² $U-B$	E_{B-V}	$B-V$	Arp ³ (C_u) & $U-B$	E_{B-V}	Notes
HD 4862	-0.02	-0.74	0.04							
HD 4976	+0.14	-0.66	0.24	0.14	-0.60	0.22	0.13	(1.03)	0.37	4
HD 5030	+0.08	-0.50	0.07	0.13	-0.53	0.16	0.08	-0.57	0.11	5
HD 5045	-0.08	-0.94	(0.10):	0.00	-0.91	...	-0.06	-0.91	(0.11):	6
HD 5277				0.14	-0.26	0.00	0.09	-0.26	0.03	
HD 5291				0.05	-0.69	0.13	-0.01	-0.67	0.03	7
HD 5980	-0.20	-0.99	(-0.04):	-0.18	-1.06	0.05				8
HD 6884				0.08	-0.64	0.15	0.00	-0.65	0.04	7
HD 7099	-0.07	-0.84	0.05	-0.04	-0.79	0.07	-0.11	-0.86	0.00	7
HD 7583	+0.16	-0.40	0.13	0.14	-0.43	0.11	0.14	-0.37	0.12	9
b							0.10	(1.06)	0.20	7
d							0.12	(1.21)	-0.06	7
e							-0.02	(1.02)	0.08	7
g							0.15	(1.21)	-0.04	7
h							-0.08	(0.97)	0.08	7
Average (excluding HD 4976, 5045, 5980)										
			0.07			0.10			0.04	
Average (only stars near NGC 371)										
			0.05			0.09			0.04	
Average of all supergiants near cepheid field and NGC 371										
					$E_{B-V} =$	0.05 (8 stars)				
						± 0.03 (standard deviation of mean)				

¹ Gascoigne (1954) aluminized reflector.² Wesselink (unpublished) silvered reflector. His results have been empirically transformed to $U-B$ using stars in common with Gascoigne.³ Arp (1958b) refractor ultraviolet system C_u in parentheses. The silvered 74-inch mirror measures, unheretofore published, have been transformed to $U-B$ by stars in common with Gascoigne.⁴ Most heavily reddened of the group, it is in the center of the densest part of SMC, far from NGC 371.⁵ Suspected variable.⁶ Very blue, near top of $U-B$, $B-V$ normal relation and therefore too uncertain a derivation of reddening.⁷ Within approximately one degree of cepheid field and NGC 371.⁸ Wolf-Rayet star.⁹ Brightest member of SMC, in south following "extension" of SMC.

plots are nevertheless given in columns 3 and 4 of Table X.

The mean absorption from the amplitude method in column 2 is $[\Delta < B >]_{\text{abs}} = 0.25$ mag. This agrees very well with the $[\Delta < B >]_{\text{abs}} = 0.23$ and 0.24 mag, derived from the average of the next two columns. Of course, this zero-point agreement results primarily from assuming that the unreddened relation is valid for the least absorbed stars in each plot. The more impressive result is the good agreement of the derived absorptions for each individual cepheid. This means that, under these assumptions, the method is a very accurate absorption indicator. The final adopted absorption is given in column 5. It has generally been taken as a mean of the preceding three columns.

It should be pointed out the usual absorption-to-reddening ratio determined in our own galaxy for photographic magnitudes is: $E_{B-V} = 4.0A_B$. In order to obtain the reddening we must divide the values in column 5 by a factor of 4. The errors are reduced by a factor 4, and the reddening comes out quite small; it is quite accurately known even when compared to the most precise photoelectric or photoelectric-spectroscopic techniques of measuring reddening.

If we assume that there is a dispersionless relationship between light-curve shape and position in the cepheid gap, as indicated by the line in Fig. 16, then the agreement between the three methods of deter-

mining the reddening in Table X shows that the average uncertainty of an adopted absorption is about ± 0.06 (av. dev.). This means that the average relative uncertainty of a single reddening determination is ± 0.015 mag. Of course we must consider the possibility that the whole group of cepheids might suffer some over-all reddening. That is, there might be no "unreddened" cepheids as assumed here.

That circumstance seems rather unlikely, however, in view of SHP II (Arp 1958 b) where it was shown that there is no more than about 0.01 mag. reddening between us and the SMC. Moreover, the blueness of some of the main-sequence stars observed in the present cepheid region around NGC 371 indicate that there are at least some unreddened stars present. Ten percent seems a quite reasonable fraction. Figure 27 shows a color-magnitude diagram of 59 stars from the cepheid area. The age-zero main sequence reddened by 0.06 mag. is indicated in the diagram and supports this general order of reddening. The regions around the present cepheid area (SHP III, V, VI, and VII) also support the conclusion that some SMC stars are unreddened and that the remainder are no more than slightly reddened.

A completely independent and specific check, however, may be obtained on the reddening value just found here for the cepheids. Table XI lists all the three-color photoelectric data available on the SMC super-

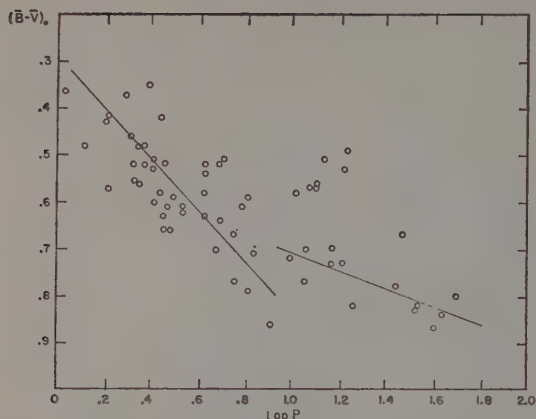


FIG. 17. The period-color relation. The unreddened colors are fitted with straight lines in order to derive approximately how much redder or bluer a cepheid is than the average cepheid at that period. The group of cepheids between $P=10$ and 16 days which are bluer than the mean are also brighter and have slower rates-of-rise to maximum light. $\bar{B}=\langle B \rangle$; $\bar{V}=\langle V \rangle$.

giants. By projecting the observed $U-B$, $B-V$ points back along the usual reddening line ($E_{B-V}=0.72E_{U-B}$) to terminate at the unreddened luminosity class I relation (Arp 1958 f; Johnson 1959) the amount of reddening can be derived for each set of observations. The last two observers in Table XI used photometric systems which admit less of the far ultraviolet than in the proper $U-B$ system (Johnson and Morgan 1953). The 74-inch silvered reflector measures were empirically transformed to $U-B$ by using stars in common with Gascoigne's. Arp's C_u measures only refer to refractor-passed near ultraviolet and the procedure for deriving reddening from them is described in SHP II (Arp 1958 b). Considering the diversity of ultraviolet systems involved, the agreement between reddening values obtained for individual stars is fairly good, particularly between columns 1 and 3. The mean reddening for the stars as a group is very closely the same for all the observers and their systems. In particular, eight of these supergiants are in the same general region as the present cepheid area. In Table XI they give an average reddening of $\langle E \rangle_{B-V}=0.05$ mag. This checks remarkably well with the reddening of $\langle E \rangle_{B-V}=0.06$ mag. just derived from the cepheids. We have, therefore, considerable confidence that our reddening values derived for the cepheids are accurate in an absolute sense.

It should be noted that the foregoing discussion clearly implies that the reddening for the SMC in general is at most only 0.01 or 0.02 mag. greater than this $\langle E \rangle_{B-V}=0.05$ or 0.06 mag. This is in contradiction to the conclusion of Feast, Thackeray, and Weselink (1958) that the average reddening of the SMC (9 stars) is $\langle E \rangle_{B-V}=0.14$ (case B). Their conclusion that the SMC stars do not show any metal weakness is also in contradiction to the conclusions of the earlier papers in this series. In this connection it should be

pointed out that diminishing metal content should cause the metallic lines in a stellar spectrum to weaken. In general this would cause the star to be assigned an earlier spectral type. When this earlier spectral type is compared to the observed color, a fictitious reddening would appear. If this circumstance exists in the SMC it would solve simultaneously the discrepancies of both metal content and reddening in the conclusions drawn from the Radcliffe spectroscopic data and the present photometric data.

19. The Unreddened Cepheid Magnitudes and Colors

Having derived the adopted values of absorption listed in column 5 of Table X, it is possible to correct each individual cepheid for reddening. These unreddened magnitudes and colors at mean light are given in the last two columns of Table X.

(a) *The Period-Color Relation.* Figure 17 shows the intrinsic period-color relation for the cepheids observed here. It is difficult to be sure of the correct mean relationship, but the lines have been drawn in Fig. 17 any way in order to give an approximate reference relation from which to measure the "blueness" or "redness" of a cepheid at a given period. For example, the break at $P=8$ days is not demanded by the points in Fig. 17 but does fit the points a little better than a smooth curve. The discrete breaks in the period-amplitude and period-asymmetry relations at this period, of course, mean there is no precedent for the relation to be continuous in the period-color plot. The seven stars between $P=10$ and 16 days, which fall very blue in Fig. 17 and appear to almost form a separate group, are stars which are not only bluer than average, but more luminous than average and have light curves with slower rates-of-rise. See Sec. 18. The position of these seven cepheids in the period-color plot mimics the position of Y Oph (Abt 1954; Kraft, to be published) but the

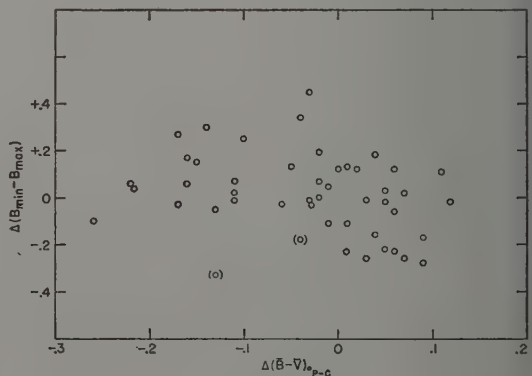


FIG. 18. The relative amplitude-color relation for the 48 longest period stars in Table XI. Within the estimated scatter of the observations (± 0.05 mag.) the diagram tends to support the conclusion that stars of largest amplitude are of intermediate color, whereas stars of smaller amplitude are either redder (the majority) or bluer (about 40% of the group). $\bar{B}=\langle B \rangle$; $\bar{V}=\langle V \rangle$.

plitude of the galactic-system cepheid is much smaller than in these seven SMC cepheids.

Of course, if we difference Eqs. (9) and (15),

$$\langle B \rangle_0 = 17.45 - 2.25 \log P \quad (10)$$

$$\langle V \rangle_0 = 17.05 - 2.47 \log P \quad (16)$$

$$\langle B \rangle_0 - \langle V \rangle_0 = 0.40 + 0.22 \log P \quad (11)$$

difference of P-L relation.

Equation (11), if actually plotted in Fig. 17, does represent the best continuous straight line fit to *all* the points. In order to get the broken line which is drawn in Fig. 17, and which ignores the group of blue points between $\log P = 1.0$ and 1.2, we would require P-L relations with discontinuities at $\log P = 0.9$ and with slightly different slopes for periods either longer or shorter than this. The departures from continuity and equality in (10) and (16) which are necessary to give a broken straight-line fit in Fig. 17, however, are only to 0.2 mag. These deviations would be negligible on a scale of the P-L relations (10) and (16). Nevertheless, if the period-color relation in Fig. 17 actually has more than one slope and is discontinuous, it implies a slight nonlinearity and discontinuity in the P-L relations.

It is now possible to read the deviation of each cepheid from the mean period-color relation of Fig. 17. These residuals are listed in the first column of Table XI and plotted in Fig. 18. There seems to be a tendency for cepheids of largest amplitude to have intermediate colors—as we expect they should. For the cepheids of smaller amplitude, the majority of them are redder and a small remainder are bluer. Figure 18 shows considerable scatter and some of the scatter is undoubtedly due to imprecisely determined reddening corrections. The error of a reddening estimate, however, is only about ± 0.015 mag. (Sec. 18). The major source of scatter in Fig. 18 is, instead, undoubtedly the observational accuracy which was estimated in Sec. 12 to be ± 0.05 in magnitude and hence somewhat more in color index. This just about accounts for the scatter in Fig. 18. In Fig. 16, of course, the abscissa is compressed by a factor of four, so the observational errors are much less a proportion of the $\Delta \langle B \rangle$. This is why the absorption can be determined from the relative amplitude-relative luminosity plot of Fig. 16 and cannot be determined very well from the relative amplitude-difference color plot of Fig. 18.

In its general form Fig. 18 does appear to confirm, however, our earliest results that the amplitude of the variables was greatest in the center and diminished towards either side of the cepheid gap. One interesting result is that the cepheids with maximum amplitude have $\Delta \langle B - V \rangle_{P-L} \approx -0.09$ mag. This comes about because the most common type of cepheid here does not have maximum amplitude and falls to the red side of the gap; therefore, the cepheids of maximum amplitude are bluer than average. It might be pointed out

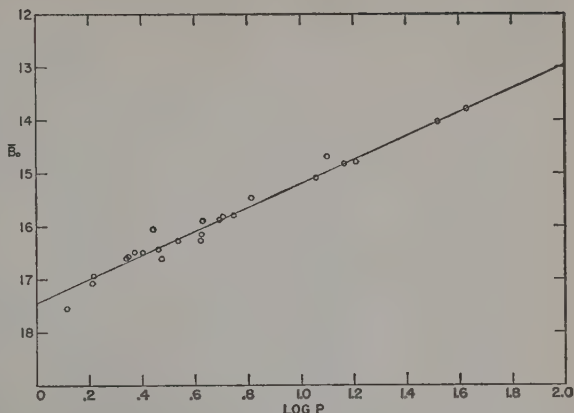


FIG. 19. The absorption-free period-luminosity relation for only cepheids with the largest amplitudes [amplitudes greater than $\Delta(B_{\min} - B_{\max}) = +0.07$ in Table IX]. $\bar{B} = \langle B \rangle$; $\langle B \rangle_0$ is from Table X.

that even if the position of the cepheid gap in the color-magnitude diagram were the same in various galaxies and star systems, if the distribution of cepheids within the gap were different, the derived mean relations would be slightly different.

(b) *The Period-Luminosity Relation.* If we plot the cepheid magnitudes corrected for reddening, $\langle B \rangle_0$, we will derive a similar relation to the plot of the uncorrected observations shown in Fig. 9. The dispersion will only be slightly less because, as we have shown in Sec. 18, the average reddening, $A_{\langle B \rangle} = 0.24$ mag., is small compared with the intrinsic dispersion in magnitude.

If we are correct in our conclusion that the cepheids near the center of the gap have the largest amplitudes, however, then we should obtain *for these cepheids alone* a much narrower period-luminosity relation. Figure 19 shows the period-luminosity diagram, corrected for reddening, for 24 of the cepheids here which have the largest relative amplitudes [$\Delta(B_{\min} - B_{\max}) \geq 0.07$ mag.]. The largest amplitude cepheids do define the narrowest P-L relation. The standard deviation of the 24 cepheids in Fig. 19 is ± 0.15 mag. The standard deviation of the remaining 38 cepheids of smaller amplitude is nearly twice as much: ± 0.28 mag.

This last circumstance is a fortunate one from the point of view of using cepheids to determine extragalactic distances. In distant galaxies the large amplitude cepheids will be the easiest to detect and measure. If enough such cepheids are available in each system, an accuracy of distance determination of ± 0.1 mag. or better can easily be obtained.

As stated in Table XI, the over-all dispersion of all 62 cepheids from the mean unreddened P-L line is ± 0.24 mag. (standard deviation). For a sample this size, the expected range is about 4.6 times the standard deviation (Hoel 1947), or the expected range 1.1 mag. in $\langle B \rangle$. This is just about the width of the P-L relation as measured here (0.9 mag. in Sec. 23). We will see

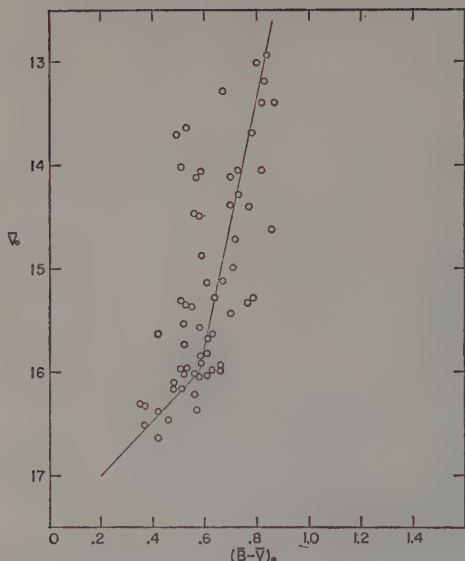


FIG. 20. The absorption and reddening corrected mean magnitudes and color indices of the cepheids. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

later how this checks with the derived slope of the constant period lines and the width of the cepheid gap in the color-magnitude diagram.

The equation of the mean unreddened P-L line in Fig. 19 is

$$\langle B \rangle_0 = 17.45 - 2.25 \log P, \quad \text{corrected for absorption.} \quad (10)$$

This equation, within the error of drawing the P-L line, is exactly the same as the apparent P-L relation in Sec. 13 when it is corrected for an average absorption of $A_B = 0.24$ mag. The residuals of the reddening corrected magnitudes from the above unreddened P-L relation, that is the intrinsic magnitude dispersions, are listed in column 2 of Table XI.

(c) *The Color-Magnitude Relation.* The unreddened, absorption-free color-magnitude diagram of the present cepheids at mean light is shown in Fig. 20. The scatter is not very much reduced from the uncorrected plot of Fig. 10 because the reddening correction is only $\langle E \rangle_{B-V} = +0.06$ mag. on the average. The increasing blueness of the faintest cepheids seems to be accentuated in the reddening-corrected plot of Fig. 20. Presumably this result is due to an accidental rearrangement of the faintest points and indicates that the mean intrinsic color of these faintest stars is uncertain from this criterion by as much as $\Delta(\langle B-V \rangle)_0 \approx 0.1$ mag. for the cepheids as faint as $V = 16.5$ mag.

20. The Lines of Constant Period in the Color-Magnitude Diagram

Figure 21 shows the logarithm of the period of each cepheid plotted in the unreddened position of that cepheid in the color-magnitude diagram. The decimal

point in $\log P$ is plotted at the exact point. Lines have been drawn connecting variables which have roughly the same period but different brightness and color. No one of these lines is very well defined, but the slopes of the whole group of eight are analyzed in Table XII. It turns out that the mean slope of the lines of constant period in the color-magnitude diagram can be determined with reasonable accuracy and is

$$\Delta(\langle B-V \rangle)_0 / \Delta \langle V \rangle_0 \big|_{P=\text{const}} = 0.68 \pm 0.10.$$

Because the reddening lines are so close in slope to the P-L line, any errors in the computed reddening of the points will have negligible effect on the derived slope of the constant period lines.

The slope observed here for the SMC cepheids is quite discordant with the slope of 0.23 computed on the basis of $P\sqrt{\rho} = \text{const}$ for the galactic cepheids (Sandage 1958). In order to discover the possible reason for this difference, we note that if cepheids obey $P\sqrt{\rho} = \text{const}$ law generally valid for vibrating systems, this leads to a functional relation between Period (P), Mass (M), effective temperature (T_e), and bolometric magnitude (M_{bol}):

$$\log P + \frac{1}{2} \log \frac{M}{M_{\odot}} + 0.3 M_{\text{bol}} - 0.3 M_{\text{bol} \odot} + 3 \log \frac{T_e}{T_{e \odot}} = \log Q. \quad (11)$$

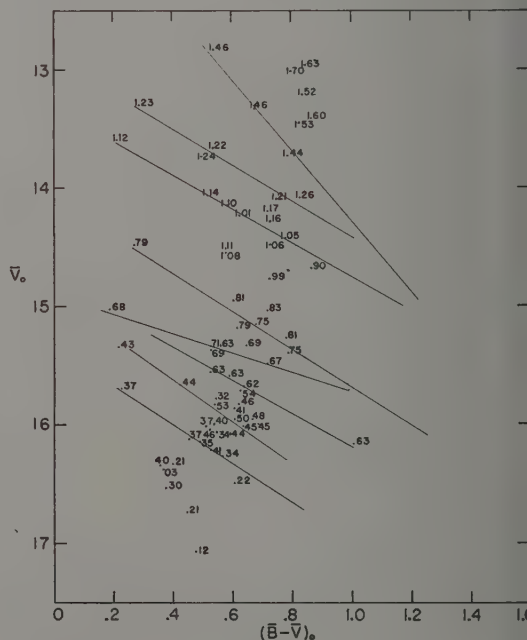


FIG. 21. The place of cepheids of different period in the intrinsic color-magnitude diagram. Log periods are plotted with the decimal point of the logarithm marking the position of the cepheid in the diagram. Lines connecting cepheids of roughly the same period are indicated. These lines are averaged in Table XII. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

period, of course, is most sensitive to the temperature, and the temperature can only be estimated by means of unreddened color indices. Therefore, the most special relation which enters the computed dependence of period on color is the color index-effective temperature scale. The points in Fig. 22 show the plot of T_e determined by Keenan and Morgan (1951) against $B-V$ colors determined by Johnson (see Arp 1959 f, 1963). Sandage fitted the points in Fig. 22 with the relation

$$T_e = 3.969 - 0.318(B-V) \quad 0.9 > B-V > 0.3, \quad (13)$$

and substitution in (12) along with assumed relations for other quantities,

$$\left. \frac{\Delta(<B-V>)_0}{\Delta V_0} \right|_{P=\text{const}} = 0.23, \quad (14)$$

Because of this large discrepancy between the observed and computed slopes of the constant-period lines, we now turn the question around and ask: What $(B-V)_0, \log T_e$ relation would the SMC cepheids require in order to predict the observed slope of the lines of constant period. At the same time we introduce a correction over the original calculation by allowing for nonhorizontal evolution.

(a) *Nonhorizontal Evolution.* The cepheids in the SMC do not appear to evolve horizontally through the diagram as the RR Lyrae cepheids appear to, and as the Galactic cepheids are assumed to. Two clusters in the SMC, NGC 458 and 330, seem to indicate cepheid evolution along a path $\Delta<V>_0 = 1/0.68\Delta(<B-V>)_0 = 0.47\Delta(<B-V>)_0$. See Fig. 28 and Sec. 26. If we assume that a cepheid loses no mass in its passage through the gap, then we can account for its diminished luminosity in Sandage's assumed constant (with $B-V$) mass-luminosity relation by substituting

$$M_{\text{bol}}' = M_{\text{bol}} + 1.47(<B-V>) + \text{const}. \quad (15)$$

This correction when substituted in (12), and using (15) again, yields

$$\left. \frac{\Delta(<B-V>)_0}{\Delta V_0} \right|_{P=\text{const}} = 0.38, \text{ using nonhorizontal evolution and (13)}. \quad (14a)$$

TABLE XII. Empirical determination of the slope of the constant-period lines in the color-magnitude diagram.

Mean log period	Slope for $P=\text{const}$ [$\Delta(<B-V>_0 - <V>_0)/\Delta<V>_0$]
1.46	0.34
1.23	0.65
1.12	0.69
0.79	0.63
0.68	1.26
0.63	0.71
0.43	0.55
0.37	0.60
Av. slope = 0.68 ± 0.10	

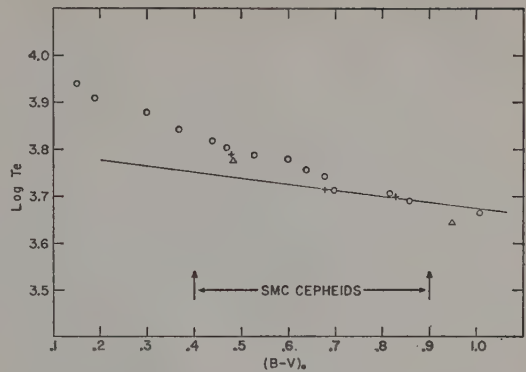


FIG. 22. Effective temperature-color index relation. Points represent T_e determinations by Keenan and Morgan (1951). Circles are luminosity class V; triangles, luminosity class III; and crosses, luminosity class I. The full line shows the relation necessary to predict the observed slope of the lines of constant period of the SMC cepheids.

This value is still far from the observed 0.68, so we proceed to calculate the $B-V, \log T_e$ dependence needed to produce constant-period lines of this slope. The relation turns out to be

$$\log T_e = \text{const} - 0.12(B-V) \quad \text{required to give constant-period line slope of 0.68.} \quad (13a)$$

This is computed with the same bolometric corrections as a function of $B-V$ as Sandage used, that is, Kuiper's bolometric corrections for main-sequence stars. It also includes the correction for nonhorizontal evolution.

Can this $\log T_e, B-V$ relation be the correct one for the present cepheids? The slope of the required relation is drawn in Fig. 22. Obviously the Keenan-Morgan points violate this slope for all luminosity classes. There have been indications, however, that the $\log T_e, B-V$ relation for galactic supergiants may be shallower than indicated by Keenan and Morgan (Kraft, to be published; Kraft *et al.* 1959). Most recently, Oke (unpublished) has made spectral scans of η Aql and computed models which will give his observed fluxes. Those model T_e 's give a slope of 0.14 for the $B-V$ dependence on $\log T_e$,

$$\log T_e = \text{const} - 0.14(B-V); \quad \text{Kraft and Oke for } \eta \text{ Aql.} \quad (13b)$$

Considering that there are still uncertainties in the bolometric corrections for these cepheids as a function of $B-V$, the agreement with (13a) is good. Put another way, the use of (13b) in (12) predicts

$$\left. \frac{\Delta(<B-V>)_0}{\Delta V_0} \right|_{P=\text{const}} = 0.60, \text{ using nonhorizontal evolution and (13b), and Oke-Kraft bolometric corrections (unpublished).} \quad (14b)$$

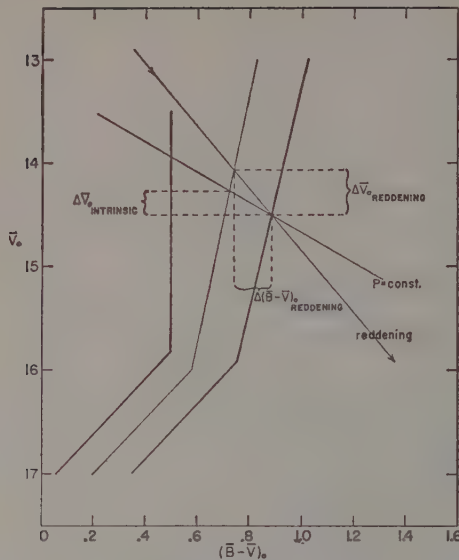


FIG. 23. Cepheid domain in the color-magnitude diagram (heavy lines). The lighter central line represents the average place of the cepheids. The direction which reddening moves a cepheid in the diagram is shown and the observed slope of a line of constant period is also illustrated. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

This computed slope of 0.60 now lies within the error of determination of the observed slope here (0.16 ± 0.10). It should be borne in mind, however, that the two slopes should not necessarily agree. Along with other differences it is possible that the $\log T_e$, $B-V$ relation is different in the SMC than it is in our own galaxy. Differences in chemical composition, for example, could affect the SMC colors through line blanketing or electron pressure.

21. The Dispersion About the Mean P-L Relation.

We proceed to test the numerical value of the slope of the lines of constant period in the color-magnitude diagram which we just derived. To do that we refer to Fig. 23. The heavy lines represent the boundaries of the cepheid domain from Fig. 20 and the central line represents the locus of the mean of the cepheids from the same figure. The line labeled $P = \text{constant}$ has the slope just derived in Sec. 20. A cepheid falling somewhere in this domain falls on some line of constant period. It has a lower than average luminosity if it falls to the right of the central line. That is, the average star of this period is bluer and brighter; therefore, this star would have a faintward dispersion from the P-L relation. The cepheids on the edge of the domain in the color-magnitude diagram define the maximum dispersion from the P-L relation. Since the whole domain of Fig. 23 is more or less filled with cepheid points, the slope of the constant-period lines and the boundaries of the cepheid domain in the color-magnitude diagram alone determine the width of the P-L relation.

The P-L relation for visual magnitudes was constructed. This P-L relation has the equation

$$\langle V \rangle_0 = 17.05 - 2.47 \log P.$$

Residuals from this line were determined and they are listed in the 4th column of Table XIII. These residuals from the visual P-L relation are plotted as points in Fig. 24. The predicted outer envelope of these residuals is drawn in. The fit is very good except for a few of the faintest cepheids where the original measures or reddening corrections are uncertain.

We see that, given the observed cepheid domain which is outlined in Fig. 23, the envelope of the dispersion is determined by the slope of the constant-period lines. If the slope were steeper, then the predicted dispersion from the P-L line would be greater. If the slope were less, the predicted dispersion would be less. The fact that the envelope of the predicted dispersion fits the points so well gives a numerical check on the derived slope of the constant-period lines as well as the entire analysis to this point.

22. The Correlation Between Deviations from the Mean P-L Relation and Deviations from the Mean Color-Magnitude Relation

It is perfectly clear from the 6th and 7th columns of Table VII that the cepheids which fall faintward from the P-L relation actually are redder than average and the cepheids which are brighter are bluer. Column 6 of Table VII measures the amount each cepheid is displaced (along a reddening line, $A_V = 3.0 E_{B-V}$) from the mean apparent color-magnitude relation drawn in Fig. 10. These residuals are plotted in Fig. 25 against residuals from the P-L line. Referring to Fig. 23, it is seen that, if the cepheids were

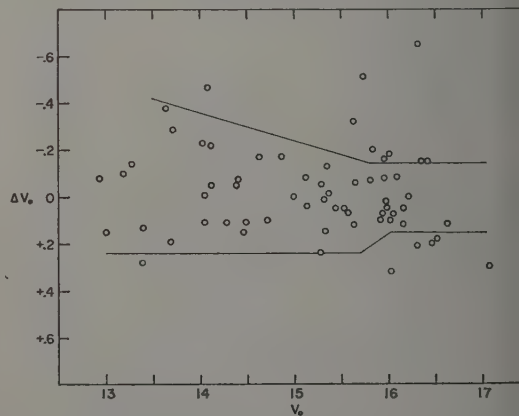


FIG. 24. Dispersion from the mean, visual P-L relation. The values of the residuals which are plotted are from column 4 of Table XIII. The envelope of this dispersion, as predicted from the slope of the lines of constant period and the domain of the cepheids in the color-magnitude diagram, is shown by the straight lines.

TABLE XIII. Residuals of reddening-corrected cepheids from mean unreddened relations.

Variable HV	$\Delta(-<V>)_0$ Period-Color	$\Delta()_0$ P-L	$\Delta(<V>)_0$ P-L	$\Delta(-<V>)_{\text{Color-Magnitude}}^*$	Variable HV	$\Delta(-<V>)_{\text{Color-Magnitude}}^*$	$\Delta()_0$ P-L	$\Delta(<V>)_0$ P-L	$\Delta(-<V>)_{\text{Color-Magnitude}}^*$
1877	-0.04	+0.17	+0.15	-0.03	1994	-0.05	+0.12	+0.07	-0.03
837	+0.01	0.00	-0.08	0.00	1793	+0.04	+0.22	+0.12	+0.02
11182	+0.05	+0.40	+0.28	+0.07	1801	+0.04	+0.03	-0.06	+0.01
2064	+0.01	+0.21	+0.01	+0.02	11206	+0.03	+0.18	+0.32	+0.03
840	+0.02	+0.01	-0.10	+0.02	1987	+0.03		+0.10	0.00
1967	-0.13	-0.20	-0.14	-0.12	1898	...	...	...	...
847	-0.01	+0.25	+0.19	+0.01	2027	+0.11	+0.22	+0.07	+0.07
1884	+0.06	+0.25	+0.11	+0.07	1974	+0.06	+0.02	+0.02	+0.02
1925	-0.26	-0.47	-0.29	-0.25	2000	+0.12	+0.12	+0.10	-0.03
1954	-0.22	-0.52	-0.38	-0.22	1809	+0.12	+0.21	+0.05	+0.07
1787	-0.02	+0.06	-0.01	-0.01	2049	+0.09	+0.17	+0.03	+0.05
843	-0.04	0.00	-0.05	-0.03	1850	-0.11	-0.04	-0.32	-0.17
1695	-0.01	+0.19	+0.11	0.00	1981	+0.05	+0.16	+0.08	+0.01
1933	-0.22	-0.35	-0.23	-0.21	1923	-0.01	+0.14	+0.12	0.00
1873	-0.17	+0.09	+0.15	-0.13	2046	+0.09	-0.09	-0.20	+0.01
1744	-0.16	-0.28	-0.22	-0.15	850	+0.02	-0.04	-0.08	-0.05
856	-0.16	+0.06	+0.11	-0.12	2021	-0.16	+0.10	+0.23	-0.07
2017	-0.02	+0.02	-0.05	-0.01	11192	+0.03	+0.12	-0.16	-0.05
2063	+0.05	+0.08	-0.07	+0.05	2002	-0.01	+0.04	+0.05	-0.02
2060	-0.13	-0.15	-0.47	-0.14	844	0.00	-0.09	-0.08	-0.04
1768	+0.01	+0.23	+0.10	+0.03	2014	+0.08	-0.09	-0.8	-0.01
1950	+0.07	+0.08	-0.17	+0.15	848	+0.09	+0.10	0.00	+0.04
1905	...	...	...	...	11199	+0.05	-0.46	-0.51	-0.07
1855	-0.03	+0.14	0.00	+0.04	2035	-0.08	+0.10	+0.18	-0.01
1945	-0.14	-0.16	-0.17	-0.07	Arp a	+0.16	-0.02	-0.15	+0.09
11193	+0.06	+0.46	+0.24	+0.14	2019	+0.01	+0.09	+0.12	+0.06
1858	-0.11	-0.08	+0.04	-0.04	11198	+0.02	-0.17	-0.15	-0.01
1892	+0.07	+0.36	+0.15	+0.12	2022	+0.13	+0.36	+0.30	+0.20
1929	-0.03	+0.03	-0.08	+0.02	11197	+0.05	-0.71	-0.65	-0.06
1903	-0.17	-0.03	+0.01	-0.11					
1827	-0.15	-0.02	0.00	-0.10	Arp b	0.00	+0.19	+0.20	+0.04
1934	-0.03	+0.04	-0.05	0.00					
1785	+0.04	+0.22	+0.05	+0.07					
1966	-0.10	-0.13	-0.13	-0.08					
1825	-0.11	-0.03	+0.05	-0.09					

* Measured along a line of constant period.

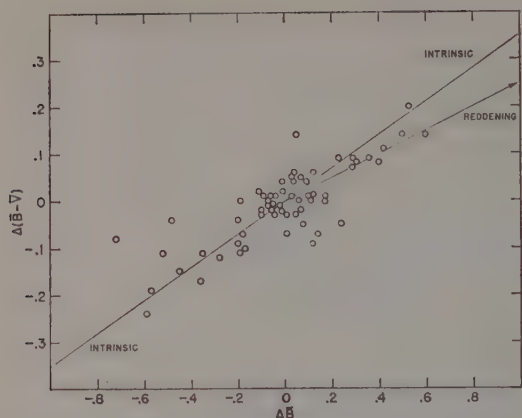


FIG. 25. The relation between how much brighter or fainter a cepheid is than the mean P-L relation ($\Delta\langle B \rangle$) and how much redder or bluer it is than the mean color-magnitude relation [$\Delta(\langle B \rangle - \langle V \rangle)$], measured along a reddening line. The intrinsic relation, from the effect of the finite cepheid domain on the $P/\rho = \text{const}$ relation, is shown. The normal reddening relation in our own Galaxy, $A_B = 4.0E_{B-V}$, is shown to have very closely the same slope. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

unreddened, for $P = \text{const}$,

$$\Delta(\langle B - V \rangle)_0 = 0.68\Delta\langle V \rangle_0 = 0.405\Delta\langle B \rangle_0, \\ P = \text{const.} \quad (17)$$

This is called the intrinsic line and plotted in Fig. 25 (allowing for a 0.88×0.405 contraction because the $\Delta(\langle B - V \rangle)$'s are measured along a reddening line rather than a $P = \text{const}$ line).

It is seen that the intrinsic line and reddening line are so close in slope that it would be impossible to use the cepheid colors to determine reddening in a practical case. These color and magnitude residuals, however, are capable of confirming our previous analysis of the reddening and also our previously derived value of the slope of the $P = \text{const}$ lines.

First notice that the observed points in Fig. 25 generally follow the intrinsic line and only the cepheids which fall the farthest below the P-L relation (large $\Delta\langle B \rangle$) approach the slope of the reddening line. This confirms our previous conclusion that most of the spread in color and magnitude of these SMC cepheids is intrinsic. At the same time, it tends also to confirm our previous derivation of a small but definite amount of reddening.

Directly below, in Fig. 26, the same plot is given with unreddened quantities. The $\Delta\langle B \rangle_0$'s are residuals of cepheids corrected for reddening from the unreddened P-L relation; the $\Delta(\langle B - V \rangle)_0$'s are residuals of cepheids corrected for reddening from the mean unreddened color-magnitude line (this time measured along a line of $P = \text{const}$). These quantities are from the 3rd and 5th columns of Table XIII. The reddening-corrected plot of Fig. 26 reduces the spread of Fig. 25 only slightly because the reddening is not large, but

now the points fit the predicted intrinsic line quite exactly.

Again this checks the whole analysis based on lines of constant period in the color-magnitude diagram and the quantitative consistency of the reddening values with the derived intrinsic values of the slope of $P = \text{const}$.

23. The Width of the P-L Relation and Color-Magnitude Domain

It is possible to simply project the intrinsic color-brightness relation at a given period, which is shown Fig. 26, onto the two coordinate axes. Projected on the $(\langle B - V \rangle)_0$ axis, the distribution of points gives a width of the cepheid gap in the color-magnitude diagram of $\Delta(\langle B - V \rangle)_0 \approx 0.3$ mag. Of course, the gap may narrow down at some magnitude levels, but in general this figure represents very well the observed width of the gap.

Projected along the $\Delta\langle B \rangle_0$ axis, the distribution gives a width for the P-L relation of $\Delta\langle B \rangle_0 \approx 0.3$ mag. This checks well with the range of 1.1 mag. computed from the standard deviation of the cepheids about the mean P-L relation.

24. SMC Cepheids with Periods near One Day and a Test for Overtones

If some of the present cepheids pulsated in higher modes than the average, then they would have a shorter than average period at a given luminosity, and hence would fall too bright with respect to the P-L relation. They would not be blue, and hence overluminous merely because they were denser than average for their period. If we take a factor of 2 for an example of such an overtone

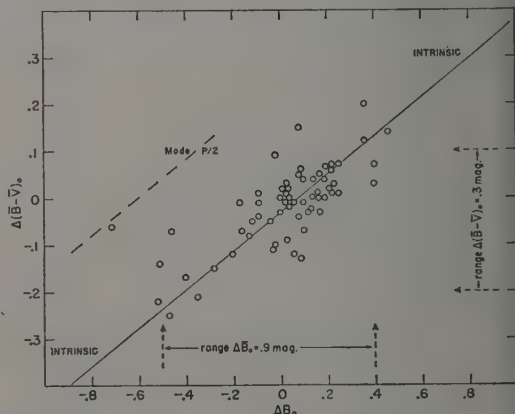


FIG. 26. Same as Fig. 25 above except all cepheids are plotted now corrected for reddening. The $\Delta(\langle B \rangle - \langle V \rangle)_0$'s are now measured along a line of constant period. The small trailing out along the reddening line in Fig. 25 is now absent, and within the scatter the points the cepheids fit exactly the predicted intrinsic relation. An overtone line, using $P/2$ as an example, is drawn as a dashed line to illustrate how the plot is used to test for presence of higher modal variation in the sample of cepheids. $\bar{B} = \langle B \rangle$; $\bar{V} = \langle V \rangle$.

the ratio of periods, then $2.25 \log 2 = 0.68$ mag. is the amount brighter that these cepheids would be than average cepheids in the lower mode. This line which the higher overtone cepheids would occupy is drawn dashed in Fig. 26.

Only one of the present cepheids is indicated to be varying near a higher mode. This is the shortest period variable HV 11197. It falls 0.7 mag. brighter in the mean P-L relation but is not very much brighter than other cepheids of the same brightness.

There appears to be a class of objects like this in the SMC. Several were reported by Dartayet and Landi Dessy (1952) and also by C. P. Gaposchkin (1958). These cepheids have periods near one day and are nearly a magnitude brighter than the P-L relation extrapolated to this period. The existence of this group, whose periods overlap the periods which we normally think of as RR Lyrae stars, was always puzzling. Their falling above the mean P-L relation about $B=17$ raised questions as to whether they are bona fide members of the SMC since they could not be tolerated as RR Lyrae stars after revision of the distance scale by Baade (1953).

Work at Cordoba (Landi Dessy 1959), however, has always indicated that the group unquestionably belongs to the SMC. The place of HV 11192 in Fig. 26 indicates that it is varying in a higher mode—roughly half the period of the brighter variables. Although this star has the shortest period of any observed here, it is by no means the faintest, and so the photometry is quite reliable. The place of HV 11192 in Fig. 21 ($\log P = 0.30$) indicates it is not very different in color or magnitude from some of the other stars around it; therefore, the difference by about a factor of 2 in period of HV 11192 compared to these other cepheids suggests strongly that it is vibrating in a higher mode. At the same time, HV 11192 and the few cepheids of about the same color and magnitude outline the bluest faint edge of the cepheid domain. It is reasonable to suppose that the mean physical conditions must change somehow to produce the onset of a higher mode. The results here suggest that it is this faint blue region of the color-magnitude diagram where the transition to the different physical conditions which produce higher modes takes place.

If this one star studied here, HV 11192, is representative of the group discovered by Dartayet and Landi Dessy, then the interpretation of this long puzzling group is simply that they are varying in higher modes. The Dartayet-Dessy group of variables could bear the same relation to the rest of the SMC cepheids as in our own Galaxy the c-type RR Lyraes bear to the a- and b-type RR Lyraes. (The cepheids do not have a particularly small amplitude for its period—see Figs. 11 and 13—but it does have a very small amplitude for its magnitude, again in analogy with RR Lyrae c types.)

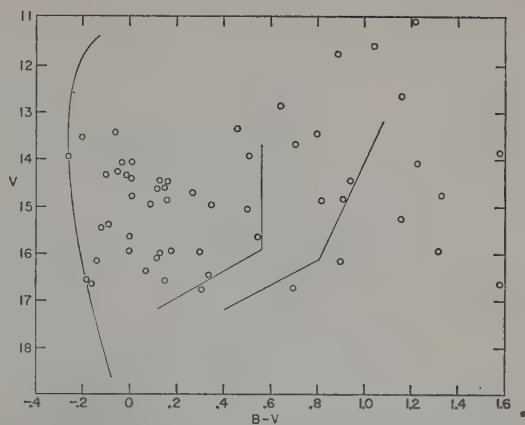


FIG. 27. The color-magnitude diagram for nonvariable stars in the cepheid field. The stars are sequence stars from Tables I and II plus miscellaneous unpublished measures. The age-zero main sequence is shown reddened by $E_{B-V} = 0.06$ mag. The cepheid domain is shown for the same amount of reddening, that is, as the domain is observed.

25. Nonvariable Stars in the Same Field as the Cepheids

Figure 27 shows the place in the color-magnitude diagram of the sequence stars listed in Tables I and II, plus some additional (unpublished) measures of stars in the region covered by Plate I. The full line on the left represents the age-zero main sequence (Sandage 1957) reddened by the $\langle E \rangle_{B-V} = 0.06$ mag. found for the cepheids in this field. The region in which the cepheids are here observed is also drawn in with the same reddening and absorption (that is, as it is observed).

Of course, Fig. 27 contains stars of differing ages. To outline the cepheid region by nonvariable stars, it is better to turn to individual clusters, such as NGC 458 (Arp 1959 a) and NGC 330 (Arp 1959 b) where nonvariable stars of the same age outline the gap more precisely at a given magnitude level. Nevertheless, Fig. 27 shows the general exclusion of nonvariable stars from the region of cepheid variability. It should be remembered when analyzing Fig. 27 that $\langle E \rangle_{B-V} = 0.06$ mag. is only an average reddening. An occasional star (estimated 10% in Sec. 18) may be bluer by 0.06 mag. than the average place of the regional lines. Also, an occasional star may be shifted far to the right by reddening. In general, however, the fit of the age-zero main sequence and the observed cepheid domain to the nonvariable stars in the field shows

- (1) The conclusions about the reddening of the cepheids are essentially correct;
- (2) The stars in the cepheid field are generally similar to the kind of stars observed in the young SMC clusters, e.g., NGC 458 and NGC 330;
- (3) There is a large population of evolved supergiants relative to main-sequence stars. This was also

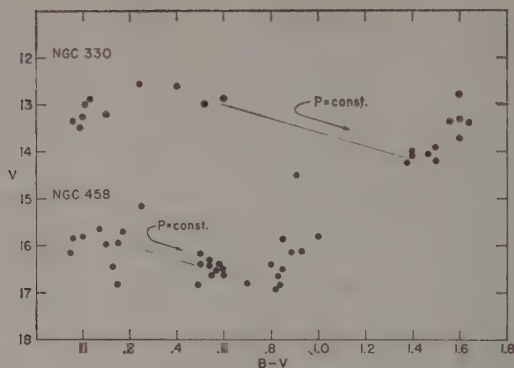


FIG. 28. The evolved giants in the color-magnitude diagrams of NGC 330 and NGC 458. The observed $P=\text{const}$ lines drawn in the gap coincide very closely with the suggested path of cepheid evolution. If no mass loss takes place, this implies that the cepheids evolve along a line of constant radius. The surface gravity and density are also constant along this path.

observed in the young clusters and quantitatively explains the impression of a ratio of cepheids to main-sequence stars that is large compared to our own galaxy.

26. Evolution of Cepheids in the Color-Magnitude Diagram

Because giant stars older than the Schönberg-Chandrasekhar limit evolve so fast relative to slightly fainter stars, the sequence they define in the color-magnitude diagram is customarily taken as defining an approximate evolutionary track. The galactic clusters which contain cepheids and evolved giants (Arp 1958 e; Arp, Sandage, and Stephens 1959) indicate roughly horizontal evolution for the cepheids between the red and blue giants. In the absence of further data, horizontal evolution has been assumed (Sandage 1958). The clusters in the SMC indicate nonhorizontal evolution for the cepheids. The difference in the evolutionary tracks between the two galaxies is probably real, but clusters in our own galaxy might simply contain too few evolved giants to show the nonhorizontal evolution.

The two young clusters in the SMC, NGC 458 and NGC 330 (SHP VI and VII), however, contain enough giants to define the cepheid gap well and to suggest a possible path of evolution through the gap. The data are shown in Fig. 28. The empirically determined constant-period lines, as derived in Sec. 20, have been drawn in. *The suggested straight line path of cepheid evolution from the blue side of the gap to the red side of the gap is very close to a period-equals-constant line.*

Now of course, if the cepheids evolve along this line, it may be for reasons which have only to do with their interior and coincidence with a $P=\text{const}$ line might be completely accidental. On the other hand, the agreement is so exact that it suggests that we may be discovering a significant clue to the nature of the pulsation.

If there is no mass loss as the cepheid evolves through the gap, then this period equals constant line is also line of constant density, a line of constant surface gravity, and a line of constant radius.

If the cepheid wanted to evolve horizontally across the gap and instead was constrained to decrease brightness along the $P=\text{const}$ line shown, then a mass loss of from 18% for NGC 330 to 11% for NGC 458 would be required (from the mass-luminosity law). The decrease in brightness in (12) however would mostly be compensated for by the change in temperature across the gap. In that case, the $\log Q$'s would have changed very little to keep the period constant across the gap.

Of course mass loss would cause a disagreement between the observed slope of the constant-period line and the slope computed from (12) on the assumption of constant mass. Although there is no evidence against mass loss in the cepheid stage, there is also no observational evidence that would suggest it. This consideration, therefore, does not seem very promising at present. Probably a more direct approach would be to calculate models evolving through the cepheid zone and determine whether they would naturally follow the path suggested by the present observations.

27. Period Changes

Period changes have been determined for a number of cepheids (see summary by Parenago, 1956). Because many of these individual cepheids show periods which sometimes decrease and sometimes increase, it has been generally concluded that these period changes are irregular perturbations and not secular variations (see Ledoux and Walraven 1958, p. 381). It would be of great interest to measure secular variations in the periods of cepheids of course, because, depending on the evolutionary track, the secular changes would be a measure of the direction and speed of evolution through the gap.

To be specific, whether a cepheid lengthens or shortens its period as it evolves through the gap depends only on whether it evolves less steeply or more steeply than a $P=\text{const}$ line. Figure 28 shows that for the SMC the $P=\text{const}$ line is so close to the line of inferred evolution that there is no way to predict whether the SMC cepheids should increase or decrease their periods. Of course, the closer the cepheid evolves along the $P=\text{const}$ line, the less will be its period change.

Now, the observations of the present group of SMC cepheids show some period changes which appear to be significant relative to their errors of determination. In order to derive period changes, the periods determined in the present investigation (last column of Table IV) were compared to the periods determined by the earlier Harvard workers, primarily as given in *Harvard Catalogue* 439 and 444. Mrs. Virginia McKibben Nail kindly ex-

ned the available Harvard record books to check the
ods given in Table IV. She notes some corrections
he listed periods and additional significant figures in
e other periods for the material derived by Craigie in
ard *Circ.* 444. These corrections and additions have
been included in Table IV, but they are minor, and
le IV seems generally to be the best representation
he available data on these periods. The amount of
od change was divided by the period and the result-
percentage period change plotted against $\log P$ in
3. The error in the derived period change was
en as equal to the error in the determination of the
period.

or periods shorter than about $\log P = 1.2$ it is seen
t about 30% of the cepheids show period changes
ch are large compared to their errors of determina-
n. Excluding three very large period changes which
outside the boundaries of the diagram, *most of the
significant period changes are negative.*

n a number of stars, all of the same kind, we would
ect the short-term perturbations to average out
en the group as a whole is considered. Any significant
dual change left would have to be considered an
age secular change for the stars in the group. The
st thorough investigation along these lines was by
serene (1952) for RR Lyrae stars in M3. There it
hed out that about as many variables were shorten-
their periods as were lengthening them. Apparently
secular term was too small to detect. The present
up of cepheids, however, is brighter and presumably
ir evolution through the gap is faster than for the
Lyrae stars, so the secular term to be detected may
considerably longer.

The average decrease in period for the cepheids which
here observed to be decreasing their periods is
out -0.4% in the time between the Harvard and the
o period determinations (taken as roughly 40 years).
is means period changes of 100% in the order of 10^4
rs. Now NGC 330 has stars about as bright as in
nd χ Per in our own galaxy. This indicates its age
f the order of 10^6 to 10^7 years. If we take the linear
sity of a sequence in the color-magnitude diagram
be roughly indicative of the time spent in a given
ion, then we might estimate the time scale of the
lving giants to be an order of magnitude less than
age of the cluster. Perhaps the number of cepheids
ly one variable known in each of NGC 330 and 458),
hence the time spent in the cepheid gap, is down
other factor of 10. Therefore, on a very approximate
ler of magnitude calculation we expect 10^4 or 10^5
rs for a cepheid to cross the gap in NGC 330. This
ght be considered within the range of our extrapolated
od changes, 100% in 10^4 years. Unfortunately, if the
heids take anything like the evolutionary path sug-
sted by Fig. 28, their total period change through the
o would be less than 100%. Perhaps the cepheids
olve more vertically through the central regions (the
servable, large amplitude regions) of the gap. An-

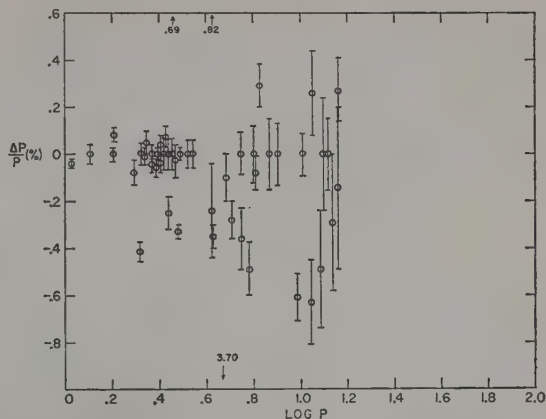


FIG. 29. Period changes. The percentage of period change over an interval of about 40 years is plotted against $\log P$. The Harvard periods are considered accurate and the estimated probable errors are taken equal to those of the determination by Arp.

other possibility is that the largest decreasing periods represent cepheids on the way back from the red-giant stage. Here we would expect their evolution to be faster and they would tend to cut the $P = \text{const}$ lines perpendicularly. The magnitude and direction of the observed period decrease would then be quite comfortable, but it would be hard to concede that as many as 25% of the cepheids here were on the return evolution path from the red-giant stage.

Therefore, there seems to be no satisfactory interpretation of the observed period changes at present. The fact that period changes which appear to be real are showing up, however, should spur efforts to more accurately determine these period changes. Particularly the periods longer than $\log P = 1.2$ could easily be determined with enough accuracy to plot in Fig. 29. For these longer periods the period changes are expected to be, and observed to be, even faster (e.g., Oosterhoff 1959, notes a 48-day cepheid in the LMC that has $\Delta P/P = +2\%/100$ years).

ACKNOWLEDGMENTS

The realization of this lengthy project of photometric research on the SMC was made possible through funds granted by the National Science Foundation and administered by Indiana University. Crucial contributions to the project were made by too many people to thank fully. In the beginning, however, encouragement by Greenstein, administration by Edmondson, and technical assistance by H. L. Johnson were particularly important. Later, continuing aid by Stoy and his staff at the Royal Observatory, and Thackeray and his staff at the Radcliffe Observatory enabled completion of the observations. The observations were analyzed with the aid of the research facilities of Indiana University and finally the facilities of Mount Wilson and Palomar Observatories.

Note added in manuscript. Working with the present data, R. P. Kraft has shown that there are non-negligible differences between $\langle B \rangle - \langle V \rangle$ and $\langle B - V \rangle$. Values of $\langle B - V \rangle$ average around 0.05 mag. redder than $\langle B \rangle - \langle V \rangle$ and the differences are largest for the largest amplitude light curves. Kraft derives

$$\frac{\Delta \langle B - V \rangle}{\Delta \langle V \rangle} \Big|_{P=\text{const}} = 0.49 \pm 0.12$$

for the slope of the constant period lines from this slightly different treatment of the present data. Which, if any, of the various ways of deriving a mean color index is physically significant will be investigated in future papers. But it should be noted that the relations involving mean color indices in this paper can be changed somewhat depending on how the mean is taken.

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Astrometric Analysis of Lalande 21185

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The variable proper motion of Lalande 21185 was discovered in 1941 from plates taken with the 24-inch Sproul telescope. Measurements and reductions of the plates over the interval 1912–1959 yield $+0''.399 \pm 0''.002$ for the relative parallax and indicate a period of $8''.0 \pm 0''.3$ with $0''.0336 \pm 0''.0024$ for the value of the semi-axis major of the photocentric orbit. Assuming a mass for the primary, appropriate for a dM2 star, a mass of the order of $0.01 \odot$ is found for the unseen companion.

INTRODUCTION

LALANDE 21185, BD+36°2147, ($10^5 57^m 9^s$ +36°38', 1900) vis. mag. 7.46 spectrum M2V, at a distance of 8.1 light-years, is an astrometric binary with a period of 8 years. The system has a space velocity of 10 km/sec. From the plates taken with the Sproul refractor, variable proper motion was discovered by Land in 1941 and announced by van de Kamp (1944). Due to the small amplitude of only 0.4", a long time elapsed before the proper period could be assigned to the perturbation. The current investigation includes the material from 1912–1914 (Miller 1914), 1921–1926 (Pitman 1928), and 1912–1940 (Land 1942), and supersedes the former results for parallax and proper motion.

MATERIAL AND MEASURES

The procedure described in previous Sproul papers has been followed (van de Kamp and Lippincott 1949). The material includes 3591 exposures on 955 plates taken on 315 nights, resulting in a total weight 783. The material is summarized by two early normal points (yearly means from 1938 on, given in Table I. The total exposure time from 1912–1926 was 15 min, 1927–38: 9 min, 1939–41: 2 min, 1942–45: $1\frac{3}{4}$ min, 1946–59: 1 min. The dependences and their yearly changes are given in Table II, which also includes the standard deviations and the position for Lal 21185 in 1940.00. The positions of the reference stars were kindly furnished by J. B. Potsky. The image of Lal 21185 was reduced to a magnitude 10.9 by the use of a rotating sector with 4.4% transmission, thus providing close magnitude compensation for the three reference stars. The measurements used in the current investigation have been made over the time interval 1937–1959 by the following persons on the Gaertner machine: Dorothy Allen, Julie E. Damkoehler, Marion Heaney, Lida Kranold, Gustav Land, Sarah L. Lippincott, Roy A. Mateer, John Merrill, Jr., Jane Piper, E. R. Reuning, Nancy G. Roman, Sara M. Smith, A. A. Strand, Marion E. Wolff, and Richard D. Pitman. Two thirds of the plates have been measured by the author. Approximately half of the plates have been measured twice, some three times. When additional years were measured, an earlier selection of

plates consisting of four or five consecutive nights were remeasured in order to check the constancy of the machine-measurer combination. In all cases averages have been used with no increase in plate weight.

PRELIMINARY REDUCTIONS

The solution for parallax and proper motion in x and y from 1912–1940 (Land 1942) provided the basis for the ephemerides until the current solution was made. The color equation amounts to $\gamma_x = +0''.0013$, $\gamma_{xz} = +0''.0003$ and $\gamma_y = +0''.0018$ (Lippincott 1957). Graphical attempts at finding an orbit which satisfied Kepler motion and represented the perturbed proper motion led to a period of $1''.14$ (van de Kamp and Lippincott 1951). The true value of the period was obscured for a long time due to the inherent errors of the same magnitude as the amplitude of the perturbation.

TABLE I. Normal points of residuals.

Epoch	Σx	Σy	$q_x t^2$ unit		$q_y t^2$ unit		orbital motion in x unit y unit		v_x unit	v_y unit
			0.0001 mm	0.0001 mm	0.0001 mm	0.0001 mm	0.0001 mm	0.0001 mm	0.0001 mm	0.0001 mm
1913.20	16	16	+55	-41	-18	0	-3	-2		
23.27	17	17	22	16	+3	+2	+10	+3		
38.20	7	13	0	0	-13	+2	0	-2		
39.15	7	13	0	0	+3	+2	+3	-7		
40.06	11	25	0	0	+14	0	-3	-1		
41.18	13	35	0	0	+14	-3	+6	+3		
42.30	11	32	0	0	+4	-4	-13	-5		
43.26	5	13	+1	-1	-6	-4	-18	+10		
44.15	25	80	1	1	-14	-2	-3	+4		
45.25	14	40	2	1	-18	0	-2	-3		
46.18	9	26	+3	-2	-14	+2	+3	+5		
47.23	18	43	4	3	+3	+2	-14	+3		
48.09	34	84	5	4	+14	0	+6	-7		
49.12	15	41	6	5	+14	-3	0	+5		
50.19	19	51	8	6	+4	-4	+9	0		
51.12	14	40	+10	-7	-4	-4	+13	+3		
52.18	11	28	11	8	-13	-2	+6	-3		
53.14	16	46	13	10	-18	0	-2	-13		
54.13	11	30	15	11	-15	+2	+8	+12		
55.20	11	32	18	13	+2	+2	-2	+8		
56.16	8	22	+20	-15	+14	0	0	-4		
57.19	6	16	23	17	+14	-3	-15	+7		
58.27	9	19	26	19	+4	-4	-3	-10		
59.17	8	21	28	21	-6	-4	-12	+6		

TABLE II. Reference system.

No.	m_{pv}	Sp.	Standard frame		Dep. 1940	$\Delta D/yr$	relative	
			x_s	y_s			μ_x	μ_y
1	11.3	K0	^{mm} -49.69	^{mm} -35.30	+0.2149	+0.00287	+0.015	-0.000
2	10.9	G0	-30.93	+47.43	0.4011	-0.00308	+0.017	+0.01
3	10.5	F8	+80.62	-12.13	0.3840	+0.00021	-0.032	-0.00
Lal 21185	(10.9)	M2	+ 7.87	+ 6.76				

tion. It now seems clear that the period is close to eight years. The plane of the orbit of the unseen companion is inclined close to 90° and is nearly parallel to the right ascension coordinate. The perturbation, therefore, exhibits itself primarily in right ascension which contributed to the delay in the proper recognition of the period. Normal point remainders provide no distinction between $P=7.8$ and 8.5 as to fitness after adjustments to the proper motion and the introduction of a term for secular acceleration.

There is no doubt as to the necessity for a quadratic term over the interval 1912-1959. Although Lal 21185 has little proper motion in right ascension, the proper motions of the reference stars cause a spurious acceleration (van de Kamp 1951). Accurate proper motions for the reference stars are not available; the field is not rich in stars, and the system of three reference stars used represents one of the largest configurations on the

Sproul program. An attempt was made to determine the relative proper motions of the reference stars from a long-exposure plate taken with the Sproul 24-inch refractor in 1912 and one in 1959 where 14 field stars were measured. Reference stars 1 and 3 nevertheless are so near the edge of the plate that the determination of their positions and hence motions are extrapolated. The resulting relative proper motions in Table II are given with reservation. The expected effect on observations is summarized in Table III.

LEAST-SQUARES SOLUTION

The conditional equations for the least-squares solution are

$$R_x = \Delta c_x + \Delta \mu_x t + q_x t^2 + (B)x + (G)y + \Delta \pi P_\alpha,$$

$$R_y = \Delta c_y + \Delta \mu_y t + q_y t^2 + (A)x + (F)y + \Delta \pi P_\delta,$$

where t is counted from 1940.0000; R_x , R_y are remainders from the ephemeris computed from the 1940 solution; q is the value for the acceleration. The geometric elements, (B) , (A) , (G) and (F) are Thiele-Innes constants multiplied by $-\alpha/a$. They yield the scale α of the photocentric orbit. Two sets of elements were considered:

$$\text{I } P=8.0, \quad T=1939.3, \quad e=0.30$$

$$\text{II } P=8.2, \quad T=1939.0, \quad e=0.20,$$

resulting in two sets of elliptic rectangular coordinates x , y , for use in the conditional equations. Solutions were made on an IBM 650 electronic computer from the weighted nightly means in x and in y , using both sets of orbital elements. The values in Table IV were for the scale factor for the 24-inch Sproul refractor, 1 mm = $18''.87$. The solution using orbital elements is adopted, keeping the other in reserve for possible use in the future. Normal points of the night residuals are given in Table I. Figures 1 and 2 show the normal points plotted with respect to the orbital motion and the quadratic term effect.

A combined solution in x and y for parallax was made with the results shown in Table V.

PARALLAX

The weight of the combined solution in x and in y for parallax is 322.40 yielding $\pm 0''.0020$ for the probable error.

TABLE III. Secular acceleration.

	x	y
perspective acceleration $-1.024 \times 10^{-6} \mu V \pi t^2$	$-0''.00002 t^2$	$-0''.00017 t^2$
spurious acceleration $-[\Delta D] t^2$	+0.00002	+0.00008
Total expected effect on observations	0.00000	-0.00009
q from least-squares solution	+0.00014 ± 1 (p.e.)	-0.00011 ± 1 (p.e.)

TABLE IV. Results of solutions in x and in y .

	Orbital elements		p.e.
	I	II	
c_x	^{mm} +7.848845	^{mm} +7.848728	
μ_x	-0.030190	-0.030182	^{mm} ± 0.000011
q_x	+0.0000077	+0.0000072	± 0.0000006
π_x	+0.02099	+0.02100	± 0.00015
c_y	+6.766978	+6.766938	
μ_y	-0.250545	-0.250544	± 0.000008
q_y	-0.0000057	-0.0000058	± 0.0000005
π_y	+0.02176	+0.02176	± 0.00022
(B)	+0.000468	+0.000362	± 0.000123
(G)	+0.001671	+0.001636	± 0.000129
(A)	+0.000332	+0.000323	± 0.000090
(F)	-0.000163	-0.000140	± 0.000096
p.e. 1_x	± 0.00235		
p.e. 1_y	± 0.00169		
$[pv_x^2]$	0.00375565	0.00375680	
$[pv_y^2]$	0.00195265	0.00199617	

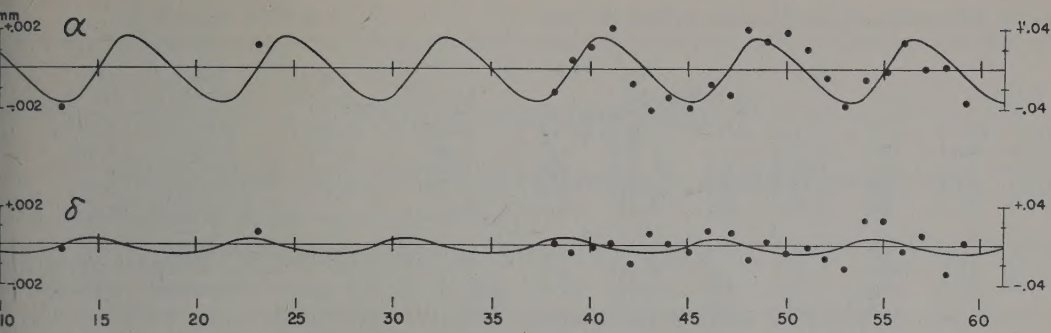


FIG. 1. Lalande 21185. Mean residuals from solution for μ , q , π represented by Kepler motion, $P=8^{\circ}0$, $T=1939.3$, $e=.30$.

Lalande 21185 is well placed with respect to the ecliptic to give an accurate parallax in declination from the present accumulation of data: $\pi_{\delta}=+0''.4106 \pm 0''.042$ (p.e.); $\pi_{\alpha}-\pi_{\delta}=-0''.00077=-0''.014 \pm 0''.005$. This difference is disappointingly large in view of the probable errors for π from the x and y solutions. A number of sources of error play a role in the parallax determination in x from which the y coordinate is free. A linear correlation of the parallax factors in right ascension with the time of the night has long been a concern of parallax observers. The spread of the instrumental configuration in x is very large shown by $\sigma_x=99 \text{ cm}^2$, $[\sigma_y]=36 \text{ cm}^2$ (Lippincott 1957) which undoubtedly contributes to the large size of the probable error of unit weight in x and may well have introduced systematic errors into the parallax determination in right ascension.

Recently we have become aware that the parallax determinations in right ascension at the Sproul Observatory are subject to systematic errors depending on the calendar years of the observations (Lippincott 1957). Differential parallax solutions grouped biennially were obtained from the night residuals from the above full solution in x and in y . The resulting differential parallax solutions are given in Table VI with the time intervals between observations. The probable errors are calculated from the weights of the parallax given for each solution combined with the probable errors of unit weight obtained from the separate x and y solutions

for the six variables. The formal probable error from the parallax solution in x must be regarded as spuriously low. No satisfactory explanation has been found for the differences in the parallax determinations in x ; the y coordinate appears free from systematic errors.

The relative parallax is reduced to absolute in the usual Sproul manner taking into account the magnitudes and the spectra of the reference stars at galactic latitude $+67^{\circ}$. The resulting absolute parallax for the

TABLE V. Results of combined solution in x and y for parallax.

	mm		p.e.
G_x	$+7.848842$		
μ_x	-0.030190	$=-0''.56968$	$\pm 0''.00020$
q_x	$+0.0000077$	$=+0.00014$	± 0.00001
G_y	$+6.767171$		
μ_y	-0.250542	$=-4.72773$	± 0.00015
q_y	-0.0000057	$=-0.00011$	± 0.00001
π	$+0.02114$	$=+0.3989$	± 0.0021
(B)	$+0.000444$	$=+0.0084$	± 0.0023
(G)	$+0.001670$	$=+0.0315$	± 0.0024
(A)	$+0.000293$	$=+0.0055$	± 0.0017
(F)	-0.000176	$=+0.0033$	± 0.0018
p.e. 1	± 0.0020	$=\pm 0.0377$	

1912-1959 Sproul material is $+0''.4039 \pm 0''.0021$. This value is compared with $+0''.384 \pm 0''.007$ (p.e.) from the three determinations: McCormick, Yerkes and Van Vleck Observatories adjusted for the Yale Catalogue precepts. Combining this value with the current Sproul value we find $+0''.402 \pm 0''.002$ for the weighted mean value and its probable error.

FIG. 2. Lalande 21185. Observed residuals after allowing for μ , π , orbital motion.

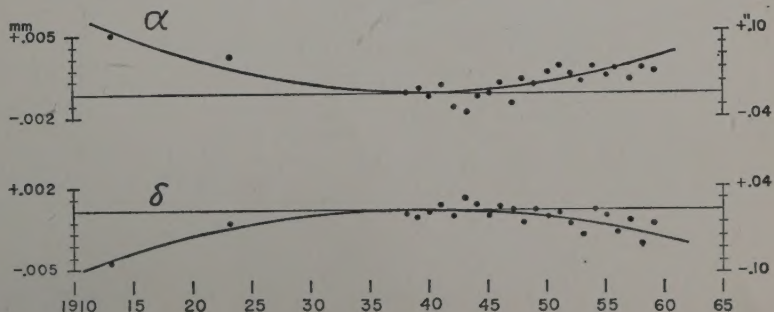


TABLE VI. Differential parallax with respect to interval of time.

Interval	Σp	$\Delta\pi_x$	p.e. unit 0.0001 mm	$\Delta\pi_y$	p.e.
1912.3-1941.4	119	- 1.0	± 3.4	- 0.3	± 3.6
42.1- 44.4	125	+ 5.9	3.5	+ 3.5	3.2
44.9- 46.4	66	+13.2	4.8	+ 1.2	3.9
46.9- 48.4	127	+17.4	3.4	+ 0.4	3.4
48.9- 50.4	92	+ 3.7	4.1	+ 2.3	3.6
51.0- 52.4	68	-46.6	5.1	- 1.8	4.3
53.0- 55.4	108	- 6.1	5.2	+ 3.7	3.8
56.1- 59.3	78	-17.9	5.3	-11.1	3.8

ORBITAL ANALYSIS

The sums of the squares of the residuals from the two solutions with different orbital elements show no significant differences. The interpretation is not dependent on a choice of one or the other orbit. In this investigation we have chosen to work with $P=8^y0$, $T=1939.3$ and $e=0.30$. The geometric elements yield

$$i=79^\circ \pm 4^\circ$$

$$\alpha = +0^m0017.8 = 0^s0336 \pm 0^s0024 = 0.083 \text{ a.u.}$$

The lower limit of the mass of the companion can be derived from

$$(B-\beta)(M_A+M_B)=0.021(M_A+M_B)^{\frac{1}{2}}$$

assuming several appropriate values for (M_A+M_B) and the luminosity function, β . Corresponding values for M_B are given in Table VII.

The absolute visual magnitude of the visible component is 10.49; using Limber's bolometric corrections for this star $M_{bol}=8.56$; its effective temperature is 3300°K (Limber 1958). The mass-luminosity relation indicates 0.33 $\odot$ as a likely value for the mass of L α 21185 A. It seems unlikely that the B component could be as bright as $10.5=m_{pv}$, $\Delta m=3$, and have gone unseen at a distance of 1". Also it seems unlikely that for $M_{pv}=13.5$ the mass could be as small as 0.03 $\odot$ in view of the value 0.04 $\odot$ determined for each component of L726-8 where $m_{pv}=15.4, 15.8$.

TABLE VII. Values for M_B .

M_A+M_B	M_B $\Delta m=\infty$	M_B $\Delta m=5$	M_B $\Delta m=3$	ma dist 196
0.250 $\odot$	0.008 $\odot$	0.011 $\odot$	0.023 $\odot$	1.0
0.300	0.009	0.012	0.027	1.1
0.400	0.011	0.015	0.035	1.2
0.500	0.013	0.018	0.043	1.3

It is concluded that $\Delta m > 3$ which leads to a mass 0.01 $\odot$ for the fainter component assuming the mass for A to be between 0.3 $\odot$ and 0.4 $\odot$.

The companion is probably extremely red; therefore observations in the infrared would decrease the mass thereby facilitating its detection. Some image scanning photoelectric device in the infrared, taking advantage of the time of greatest separation and the position angle might yield the positive results needed for a rigorous mass determination.

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